May 26

Wednesday, May 26, 2021 2:42 PM

Comp Sci 21

1. Stable Matchings
2. Gale-Shapely

Announcements

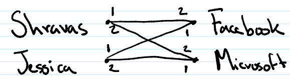
- Homework 7, last homework
- Final, no make-up

Stable matching - n jobs → ranked preferences; n applicants → ranked preferences

How do we assign applicants to jobs?

Residency matching

NYC public school, Catholic school, Chicago, high school



Rogue

~~Shravas — Facebook~~

~~Jessica — Microsoft~~

Def. Rogue couple - a job, applicant pair, both job + applicant - prefer each other over their current assignment.

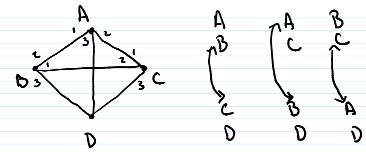
Def. Stable matching - a perfect w/ no rogue couples.

Shravas —— Microsoft → Stable matching

Jessica —— Facebook

Thm. There always exists a stable matching between jobs + applicants.

Gale-Shapely algorithm - used to construct a stable matching



rogue

No stable matching

(bi-partite partition vertices into two part, edges go from one part to another)

Proof. (Gale-Shapely)

1. Each applicant interviews at their favorite job.
2. If a job more than one applicant, reject all but their favorite.
3. If applicant gets rejected, remove favorite job from list.
4. Repeat 1-3 until every job has at most one applicant.

Day 1 - Only Shravas interviews Facebook

Day 2 - Both Shravas and Jessica interview at Facebook. (Jessica was rejected from favorite on day 1). Facebook prefers Jessica, rejects Shravas.

1. Procedure ends
2. Everyone gets matched.
3. No rogue couples

1. Every applicant has a list of n jobs. If some job has more than one applicant, the applicant crosses off a job. Jobs are never added to lists. Total # of jobs is strictly decreasing.

P = for each job j and applicant a, if j is crossed off a's list, j prefers another applicant b to a, s.t. j is b's favorite (not crossed off).

2. Assume that applicant a did not get a job. There is some job j that is unfilled, and crossed off a's list. By property P, j has an interested applicant, contradicts fact that j is unfilled.

3. Assume job j + applicant a are rogue. Either j is crossed off a's list (Case 1) or not (Case 2).

Case 1 - by property P, j prefers another interested applicant to a, j + a can't be rogue.

Case 2 - a prefers their job to j, a + j can't be rogue.

$\square$

Def. Matching a + j is possible if there exists a stable matching w/ a + j matched.

Thm. Each applicant gets favorite possible job.

Proof. Assume that some applicant crosses off favorite possible job. Assume applicant a did this first, favorite possible job is j. j prefers another applicant a' to a, and a' prefers j to all possible jobs (otherwise a' would have been first, not a).

Consider matching where a + j are matched. a' + j are a rogue couple, so the matching is not stable.

Theorem. Each job gets least favorite possible applicant.

Proof. Use contradiction, assume that job j is matched with applicant a, but a' is job j's least favorite possible applicants. By previous theorem, j is a's favorite possible job.

If a' + j are matched, then a + j are a rogue couple, the matching is not stable, which is a contradiction.