April 14

Wednesday, April 14, 2021 12:54 PM

Comp Sci 212
1. Counting Rules

Announcements
- HW1, due 11:59 pm
- HW2 out

Counting
- Bijection rule - If $f: A \to B$ is a bijection (every $a \in A$ appears in exactly one pair, every $b \in B$ appears in exactly one pair), then

$|A| = |B|$

Ex. Donut store

3 types of donuts - C, V, S

How many ways are there to buy 8 donuts? (multi-sets)

A = set of ways to buy 8 donuts
set of multisets

B = set of sequences of 8 0's and 2 1's

$\{C, C, C, S, S, S, S, S\} \in A$

$(0, 0, 0, 1, 1, 0, 0, 0, 0, 0) \in B$

$f: A \to B$, $f(a) =$ 0 # of C's in a

1

0 # of V's

1

0 # of S's

$f(\{C, V, V, V, V, S, S, S\}) =$

$(0, 1, 0, 0, 0, 0, 1, 0, 0, 0)$

Thm. f is a bijection

Proof. f is clearly total and a function (total: for every $a \in$ domain appears in at least 1 pair)

Let $s \in B$ $s = s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9 s_{10}$

Let $s_i = 1$, $s_j = 1$ $i, j \in [10]$ $i < j$, rest 0.

$f^{-1}(s) = \{a \mid f(a) = s\}$

$= \{\{\underbrace{C, C, .., C}_{i-1 \text{ times}}, \underbrace{V, .., V}_{j-i-1 \text{ times}}, \underbrace{S, .., S}_{10-j \text{ times}}\}\}$

by definition of f.

$|f^{-1}(s)| = 1$, injective and surjective $\square$

Corollary $|A| = |B|$.

Product Rule - $|A \times B| = |A| \cdot |B|$

Ex. 3 course meal

5 appetizers A

10 entrees B

7 desserts C

meals $= |A \times B \times C| = |A| \cdot |B| \cdot |C| = 5 \cdot 10 \cdot 7 = 350$

of strings in $\{0,1\}^n$ is $|\{0,1\}^n| = |\{0,1\}|^n = 2^n$
(# of strings of length n w/ 0's & 1's)

Sum rule. If $A \cap B = \phi$, $|A \cup B| = |A| + |B|$

Ex. I have 4 sweaters from H+M A

2 Uniqlo B

1 dad C

$|A \cup B \cup C| = |A| + |B| + |C| = 7$

Thm. There are 49,950 odd numbers from 100 to $100{,}000 = A = \{x \mid x \text{ is odd}, 100 \le x \le 100000\}$.

Proof. $A_1 = \{x \mid x \text{ is odd}, 100 \le x < 1000\}$

$A_2 = \{x \mid x \text{ is odd } 1000 \le x \le 10000\}$

$A_3 = \{x \mid x \text{ is odd } 10000 \le x \le 100000\}$

$A = A_1 \cup A_2 \cup A_3$, $|A| = |A_1| + |A_2| + |A_3|$ by sum rule

$D_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_3 = \{1, 3, 5, 7, 9\}$

$D_2 \to$ 1st digit can't be 0

$D_1 \to$ middle digits can be anything

$D_3 \to$ last digit has to be odd

$|A_1| = |D_2 \times D_1 \times D_3| = 9 \cdot 10 \cdot 5 = 450$ by product rule

$|A_2| = |D_2 \times D_1 \times D_1 \times D_3| = 9 \cdot 10 \cdot 10 \cdot 5 = 4500$

$|A_3| = |D_2 \times D_1 \times D_1 \times D_1 \times D_3| = 9 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 45000$

$D_1 \times D_1 \times D_3$ includes 015

Generalized Product Rule

Ex. Race w/ n runners - set R = {Brando, ... }

How many ways are there to assign 1st, 2nd, 3rd? = $n_1 = n$, $n_2 = n-1$, $n_3 = n-2$ total $= n_1 \cdot n_2 \cdot n_3 = n(n-1)(n-2)$

Try bijection from ways $\to R \times R \times R$

(Brando, Brando, Brando) $\in R \times R \times R$

Generalized Product Rule - If S is a set/collection of length-k sequences $s \in S$ $s = (s_1, s_2, ..., s_k)$

- n_1 = # of possibilities for s_1
- n_2 = # of possibilities for s_2, given s_1

⋮

- n_k = # of possibilities for s_k given $s_1, s_2, \ldots s_{k-1}$

$|S| = n_1 \cdot n_2 \cdot \ldots \cdot n_k$