April 9

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Comp Sci 212
1. Sets
2. Sequences
3. Relations

Announcements
- Link to live notes on Canvas

Def. A set is a collection of objects.
each appearing at most once (multi-sets)
unordered (sequence).

$[n] = \{1, 2, \ldots, n\}$

$A \subseteq B$ - containment (sets)

$a \in B$ - element of (element)

| - such that (:)

Ex. $\{\frac{p}{q} \mid p, q \in \mathbb{Z}, q \neq 0\} = \mathbb{Q}$ (rational numbers)

$\{2^k \mid k \in \mathbb{N}\} = \{1, 2, 4, 8, \ldots\}$ - powers of 2 ($\mathbb{N}$: non-negative integer)

$\{a^2 \mid a \in \mathbb{Z}\}$ - perfect squares ($\mathbb{Z}$: all integers)

$\{a \mid a \in \mathbb{R}, a^2 < 0\} = \emptyset$ empty

$P(A)$ = set (collection) of subsets of A (power set)
$= \{a \mid a \subseteq A\}$

$P([3]) = P(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

$\{s \mid s \subseteq [n], 1 \in s\}$ = set collection of all subsets of
$\{s \mid s \in P([n]), 1 \in s\}$ $[n]$ that contain 1.

If $n = 3$ - $\{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$

Def. If A is finite, $|A|$ = # of elements in A.

$S_r = \{s \mid s \subseteq [n], |s| = r\}$, $n = 3$, $r = 2$
$= \{\{1,2\}, \{1,3\}, \{2,3\}\}$.

$E = \{s \mid s \in S_r, 1 \in s\} = \{\{1,2\}, \{1,3\}\}$.
$n = 3, r = 2$

$A \subseteq B$ vs. $A \subset B$

$A = B$ is okay $A = B$ not okay

Proof. $A \subseteq B \longrightarrow$ if $a \in A$ then $a \in B$
$A = B \longrightarrow a \in A$ iff $a \in B$
$A = \emptyset \longrightarrow$ there does not exist elements in A
contradiction.

Thm. $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$ (actually equality)

Proof. If $a \in A \cup (B \cap C)$, then $a \in A$ or $a \in (B \cap C)$.
Use cases. Case 1 Case 2

Case 1 - If $a \in A$ then $a \in A \cup B$ and $a \in A \cup C$ so therefore $a \in (A \cup B) \cap (A \cup C)$.

Case 2 - If $a \in (B \cap C)$ then $a \in B$ and therefore $a \in A \cup B$. Also $a \in C$ and $a \in A \cup C$. Finally, $a \in (A \cup B) \cap (A \cup C)$.

□

Sequences Ex. $(1, 2, 3, 4) \neq (2, 1, 4, 3)$
$(1, 1, 1, 1)$

If s is a sequence, s_i = i^{th} element of the sequence

$A \times B = \{(a, b) \mid a \in A, b \in B\}$ (Cartesian product; (a,b): sequences)

$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i\}$.

If $A_1 = A_2 = \cdots = A_n$,
$A_1 \times A_2 \times \cdots \times A_n = A^n$

Ex. $\{0,1\}^n$ = set of all binary strings of length n

$\{0,1\}^3 = \{(0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$.

$\{s \mid s \in \{0,1\}^3, s_1 = 1\} = \{(1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$.

$s_2 = 1$

$\{s \mid s \in \{0,1\}^n$, s does not have consecutive 1s$\}$
or s starts w/ 1

Def. Relation is a subset of $A \times B$ (domain, codomain)

Ex. $\{(a,b) \mid a, b \in \mathbb{R}, b^2 = a\} \subseteq \mathbb{R} \times \mathbb{R}$
square root

If every $a \in A$ appears in at most one pair - function $\rightarrow$ $f(a)$ be corresponding element, $(a, f(a)) \in$ relation
at least - total

If every $b \in B$ appears in at most one pair - injective
at least surjective

Ex. $\{(a,b) \mid a \in [n], b \in [2n], b = 2a\} \subseteq [n] \times [2n]$
$\{(1,2), (2,4), \ldots\}$ $(a, 2a)$

$f: [n] \rightarrow [2n]$, $f(a) = 2a$

f is injective, $f(x) = f(y) \rightarrow 2x = 2y \rightarrow x = y$.
f is not surjective, $f^{-1}(b) = \emptyset$ if b is odd.
$f^{-1}(1) = \emptyset$

$f: A \rightarrow B$, function,
subset of $A \times B$.

$f: [n] \rightarrow [n]$
$f(a) = a$

$f^{-1}(b) =$

If every $b \in B$ appears in at most one pair - injective
at least surjective

$f: A \to B$, function,
subset of $A \times B$.

All four - bijection.

Inverse image $f^{-1}(b) = \{a \mid f(a) = b\}$ (function)

$f^{-1}(C) = \{a \mid f(a) \in C\}$

Proofs - Injection $f(x) = f(y)$ implies $x = y$.
$|f^{-1}(b)| \leq 1$ for all $b \in B$

Surjection $|f^{-1}(b)| \geq 1$ for all $b \in B$.

$f: [n] \to [n]$
$f(a) = a$

$f^{-1}(b) =$