March 31

Wednesday, March 31, 2021 2:25 PM

Comp Sci 212

1. Direct Proofs
2. Cases
3. Contrapositive
4. Iff

Announcements

- 1st discussion 4/1, 4/2, LaTeX tutorial
- TA/PM Office hours start 4/7

Direct proof - sequence of implications that combine

Theorem (Arithmetic-Mean Geometric Mean)

If $a, b \geq 0$, then

$\frac{a+b}{2} \geq \sqrt{a \cdot b}$

Ex. $a=2, b=2$ $\quad \frac{2+2}{2} = 2$ $\quad \sqrt{2 \cdot 2} = 2$

$a=1, b=9$ $\quad \frac{9+1}{2} = 5$ $\quad \sqrt{9 \cdot 1} = 3$

If $\frac{a+b}{2} \geq \sqrt{a \cdot b}$ $\quad a+b \geq 2\sqrt{ab}$

$(a^2+2ab+b^2) \geq 4ab$

$a^2-2ab+b^2 \geq 0$, $(a-b)^2 \geq 0$.

Proof. We know that $(a-b)^2 \geq 0$, or $a^2-2ab+b^2 \geq 0$. By adding $4ab$ to both sides this implies $a^2+2ab+b^2 \geq 4ab$, or $(a+b)^2 \geq 4ab$. Because $a, b \geq 0$, we can take the $\sqrt{\ }$ of both sides, to get $a+b \geq 2 \cdot \sqrt{ab}$.

Cases. If P then Q

If P then P_1 or P_2

If P_1 then Q

If P_2 then Q

Theorem. If $a+b \geq 16$, then either $a \geq 8$ or $b \geq 8$.

$a=17, b=1$ $\quad a=1, b=17$ $\quad a=9, b=9$

Proof. Use cases. Either $a \geq 8$ or $a < 8$.

Case 1: If $a \geq 8$, done.

Case 2: If $a < 8$, then $-a > -8$, because $a+b \geq 16$, so therefore $b \geq 8$.

□

Theorem. In every group of 6 people, there are either 3 mutual friends, or 3 mutual strangers.

Proof. Use cases. Pick person x. Either x has at least 3 friends or at most 2.

Case 1 $\quad$ Case 2

Case 1. All of x's friends are mutual strangers, or not. → Case 1b $\quad$ Case 1a

Case 1a: 3 mutual strangers.

Case 1b: y & z are friends w/ x, and each other, 3 mutual friends.

Case 2: All of the people x doesn't know are mutual friends or not.

Case 2a $\quad$ Case 2b

Case 2a: 3 mutual friends

Case 2b: y & z are strangers w/ each other & with x, 3 mutual strangers.

□

Alternate proof.

x has exactly 5 friends

x has exactly 4 friends

⋮

x has no friends.

Contrapositive - Instead of If P then Q, If not Q then not P

Theorem. If $a+b \geq 16$, then $a \geq 8$ or $b \geq 8$.

Proof. Use contrapositive. If $a < 8$ and $b < 8$, then $a+b < 16$.

□

Theorem. If $r \geq 0$ and is irrational, so is $\sqrt{r}$.

$r \neq \frac{m}{n}$ for any integer m & n.

Proof. Use contrapositive.

If $\sqrt{r}$ is rational, then $\sqrt{r} = \frac{m}{n}$ for integers m & n. Therefore $r = \frac{m^2}{n^2}$, m^2 & n^2 are both integers, so r is rational.

□

Converse - If Q then P → not always true

If $0 \leq x \leq 2$, then $-x^3+4x+1 \geq 0$ → proved yesterday

converse is not true, $x = -0.001$.

If $-x^3+4x+1 \geq 0$, then $0 \leq x \leq 2$.