March 30

Tuesday, March 30, 2021 2:06 PM

Comp Sci 212
1. Administrivia
2. Propositions
3. Direct Proofs

212	Future
Induction	Analyse recursive algorithms
Combinatorics (Math of counting)	Lower bounds. Can you sort faster than MergeSort?
Probability	Randomized Algorithms
Graph Theory 	Graph Algorithms
Linear Algebra	Everything

Def. Proposition is a statement that is true or false.

Example $2+2=4$ (true)

$2+2=5$ (false)

Example For all differentiable functions f, g

$$\frac{d}{dx} f(x)\cdot g(x) = \left(\frac{d}{dx} f(x)\right) g(x) + f(x)\left(\frac{d}{dx} g(x)\right)$$

Product Rule (true)

$$\frac{d}{dx} f(x)\cdot g(x) = \left(\frac{d}{dx} f(x)\right)\left(\frac{d}{dx} g(x)\right) \quad \text{(false)}$$

Ex. For every non-negative integer n,

$p(n) = n^2+n+41$ is prime $\hookrightarrow$ only divisible by 1 + itself (false)

$p(1) = 43$
$p(2) = 47$
$p(3) = 53$ } all prime
⋮
$p(20) = 461$
⋮
$p(39) = 1601$
$p(40) = 40^2+40+41 = 41^2$ - not prime

Example. If a, b, c, d are positive integers
$a^4+b^4+c^4 \neq d^4$ (false)
[Euler's conjecture]

$a = 95{,}800$ $b = 217{,}519$ $c = 414{,}560$ $d = 422{,}481$
(Noam Elkies)

Def. Predicate a proposition that depends on one or more variables.

Example n is a perfect square
$\hookrightarrow$ variable

P(n) = "n is a perfect square"

P(4), P(9) = True P(5), P(7) = False

Def. Axiom proposition accepted as being true (does not need to be proved)

This class - use best judgement

Theorems, Lemma, Claim - types of propositions

Implication - a statement of the form "If P then Q"

Direct proof - a sequence of implications, each that do not have to be proved $\to$ axiom, $\to$ we already proved it, that combine to form its own implication.

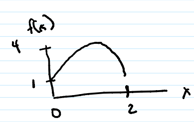
"If P_1 then P_2. Therefore P_3. Therefore, P_4"

If P_1 then P_4.

Theorem. If $0 \le x \le 2$, then $f(x) = -x^3+4x+1 \ge 0$.

Scratch work -

x	f(x)
0	1
0.5	2.875
1	4
1.5	3.625
2	1



$-x^3+4x = -x(x-2)(x+2) \ge 0$
neg neg pos

Proof. If $0 \le x \le 2$, then $-x \le 0$, $x-2 \le 0$, $x+2 \ge 0$.
Therefore, $-x(x-2)(x+2) \ge 0$.
Therefore, $-x^3+4x \ge 0$.
Therefore, $-x^3+4x+1 \ge 0$.

$\square$

Examples are not proofs.

Counterexamples are proofs.