

Boolean Functions

1. Summary

- $(2, q)$, $(p, 2)$ - hypercontractivity

2. Q+A

3. Problems

Def. $(2, q, p)$ hypercontractive if $-X$

$$\|a + pX\|_q \leq \|a + X\|_2 \quad \forall a \in \mathbb{R}$$

(p, q, p) - hypercontractive if

$$\|a + pX\|_p \leq \|a + X\|_q$$

Ex. uniform over $\{-1, 1\}$

is $(2, 4, \frac{1}{15})$ -hc

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$(2, q, \frac{1}{15})$ -hc, $q \geq 2$, even

$$\downarrow$$

$$\|T_{\frac{1}{\sqrt{q-1}}} f\|_q \leq \|f\|_2 \quad f \text{ is a function over } \{-1, 1\}$$

Thm. $\|T_{\frac{1}{\sqrt{p-1}}} f\|_2 \leq \|f\|_p \quad 1 \leq p \leq 2$ ←

$$\downarrow$$

$$\|T_{\frac{1}{\sqrt{q-1}}} f\|_q \leq \|f\|_2 \text{ for all } q \geq 2.$$

Generalizes to $f: \{-1, 1\}^n \rightarrow \mathbb{R}$ by induction

Problem 1 - Part a → uses proof from section 9.5
part b → uses dual norm tricks

Problem 2 - (q, q, p) hypercontractivity

Problem 3 - (p, q) hypercontractivity