March 9

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Boolean Functions

1. Summary
   - Dictator vs. No-notables
2. Q+A
3. Problems

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Announcements
- Week 8 Problems due date
- Website videos (2012 class) have better

Def. $\mathrm{Inf}_i^{(\rho)}[f] = \mathrm{Stab}_\rho[D_i f]$

$$I^{(\rho)}[f] = \sum_{i=1}^n \mathrm{Inf}_i^{(\rho)}[f]$$

Thm. $f: \{-1,1\}^n \to \mathbb{R}$, $\mathrm{Var}[f] \le 1$

$0 < \varepsilon, \delta < 1$

$J = \{ i \in [n] : \mathrm{Inf}_i^{(1-\delta)}[f] \ge \varepsilon \}$ $(\varepsilon, \delta)$-notable coordinates

Then $|J| \le \frac{1}{\varepsilon\delta}$

Proof. $\mathbb{I}^{(1-\delta)}[f] \le \frac{1}{\delta}$

If $J = \phi$, f is quasirandom

Dictator vs. No-notables

Let $\Psi$ be set of predicates over $\Omega = \{-1,1\}$

Let $0 < \alpha < \beta < 1$

Local tester for f

- $\Pr[\text{test accepts } f]$ $\ge \beta$ if f is a dictator

  $\le \alpha + \lambda(\varepsilon)$ $\to 0$ as $\varepsilon \to 0$ if f has no $(\varepsilon, \varepsilon)$ notable coordinates

- Tester use predicates from $\Psi$

Thm. If there such a test, it is UG-hard to $(\alpha+\delta, \beta-\delta)$-approximate Max-CSP$(\Psi)$ for all $\delta > 0$

Hastad $\forall\, 0 < \delta < \frac{1}{8}$, there exists a $(\frac{7}{8}+\delta, 1-\delta)$-test using OR

$\forall\, 0 < \delta < \frac{1}{2}$, $\exists\, (\frac{1}{2}, 1-\delta)$-test using linear equations

Test - choose $x, y$ independently at random $\in \mathbb{F}_2^n$, $b \in \mathbb{F}_2$

$z = b \cdot (x \circ y)$, $z' = N_{1-\delta}(z)$

Accept if $f(x) + f(y) + f(z') = b$

Problem 1 - What happens if you apply the test $T_\rho f$ (refer to Problem 2)

Problem 2 - $\mathrm{Inf}_i^{(1-\delta)}[f]$ vs. $\mathrm{Inf}_i[f]$ (uses problem 1)

Problem 3 - tests that use linear equations $\updownarrow$ test that use or

- There is a subexponential time for UG for certain Arora-Barak-Steurer
- A lot of cases, the problem is NP-hard Khot-Minzer-Safra

$x, y, z$

accept if $f(x) \vee f(y) \vee f(z) = 0/1$