March 4

Thursday, March 4, 2021 1:41 PM

Boolean Functions

1. Summary
   - CSPs
   - Hardness of Approximation
   - Max-Cut
2. Q&A
3. Problems

CSPs - Max-3SAT

Max-Cut

Max-E3Lin

def. domain Ω predicate Ψ

def. instance of Max-CSP list of constraints over variables V

def. assignment $F: V \to \Omega$

$\mathrm{Val}_{\mathcal{P}}(F)$ = fraction of constraints satisfied.

CSP $\longleftrightarrow$ String testers

Max-E3Lin $\leftrightarrow$ BLR

def. (α, β)-approximation algorithm,

Know that exists assignment s.t. $\mathrm{Val}_{\mathcal{P}}(F) \geq \beta$, output assignment. $\mathrm{Val}_{\mathcal{P}}(F) \geq \alpha$.

Thm. PCP Thm. $\exists \delta_0$ s.t. $(1-\delta_0, 1)$-approximating Max-E3SAT is NP-hard.

Thm. PCPP Thm implies PCP Thm reduction from circuit-SAT.

$\delta_0 < \frac{1}{8}$, random assignment $\frac{7}{8}$-approximation

Hastad - NP-hard to $(\frac{7}{8}+\varepsilon, 1)$-approximate Max-E3SAT

NP-hard to $(\frac{1}{2}+\varepsilon, 1-\varepsilon)$-approximate Max-E3Lin.

Max-Cut

Given $G=(V,E)$ what is the partition into V_1, V_2 that maximizes # of edges from V_1 to V_2.

Maximize $\sum_{(u,v)\in E} (x_u - x_v)^2$ s.t. $|x_v| = 1$ for all $v \in V$

$V_1 = \{v \mid x_v = 1\}$ $V_2 = \{v \mid x_v = -1\}$

NP-hard

Randomized - $\frac{1}{2}$-approximation

Hastad - NP-hard to 0.941-approximate Max-Cut

SDP-Relaxation (GW)

Max $\sum_{(u,v)\in E} \langle x_u - x_v, x_u - x_v \rangle$ s.t. $\|x_v\|_2 = 1$ for all $v \in V$

$x_v \in \mathbb{R}^d$

Random $\theta \in \mathbb{R}^d$, Polynomial w/ SDP-Solver

$V_1 = \{v \mid \langle x_v, \theta \rangle \geq 0\}$ $V_2 = \{v \mid \langle x_v, \theta \rangle < 0\}$.

$\Pr[v,u \text{ separated}] \geq \underline{.87856} \langle x_u - x_v, x_u - x_v \rangle$ (geometry).

Khot, Kindler, Mossel, O'Donnell - Unique-games-hard to $.87856$-approximate Max-Cut

Problem 1 - decision vs. search for approximating

Problem 2 - analyzing randomized algorithms $\downarrow$ only one that needs Fourier analysis

Problem 3 - from randomized $\downarrow$ deterministic

Room - Problem: