March 2

Tuesday, March 2, 2021 1:46 PM

Boolean Functions

1. Summary
   - Local testers/PCPP's
2. Q+A
3. Problems

Announcements
- March 11 Lecture changed
- Ryan's website for the course has better quality videos (Campuswire)

Def. $\mathcal{C} \subseteq \{0,1\}^n$ a property

local tester w/ rejection rate λ if

$w \in \mathcal{C} \to$ accept w/ prob 1

$\text{dist}(w, \mathcal{C}) > \epsilon \longrightarrow$ reject w/ prob. $\lambda\epsilon$

Thm. Let $\mathcal{C} \subseteq \{0,1\}^n$, $\exists$ a 3-query PCPP system for $\mathcal{C}$ w/ rejection rate .002, length 2^{2^n}

Proof. Interpret each $x \in \mathcal{C}$ as a number in $[2^n]$

$\mathcal{C} \subseteq [2^n]$, turn each $x \in \mathcal{C}$ into a dictator, do dictator testing

def. of $\mathcal{C}$ is embedded into test

$x_1 \neq x_2$

$x_1, x_2 \in \{0,1\}^n$

$\text{dist}(x_1, x_2) \geq \frac{1}{n}$

distance between dictator versions is $\frac{1}{2}$

r-query length-ℓ PCPP system w/ rej. λ if

$\exists$ string tester T operating on $(w, \Pi) \in \{0,1\}^{n+\ell}$

If $w \in \mathcal{C}$, exists Π s.t. T accepts

If $\text{dist}(w, \mathcal{C}) \geq \epsilon$, then for every Π, T rejects w. prob $\geq \lambda\epsilon$

an anlogue NP, P

- $\mathcal{C} = \{w \in \{0,1\}^n$ s.t. w has an odd # of 1's$\}$

  $\llcorner$ any local tester requires n queries

  3 query PCPP system

  $w_1, w_2, \ldots, w_n, w_1+w_2, w_1+w_2+w_3, \ldots, w_1+w_2+\cdots+w_n$

  $\Pi_1 \quad \Pi_2 \quad \cdots \quad \Pi_{n-1}$

def. PCPP reduction - algorith which outputs a description of the test. C circuit reps. test

Thm. There exists a PCPP reduction w/ proof length $2^{\text{poly}(\text{size}(C))}$

Thm. PCP poly(size(C))

Thm. size(C) poly(log(size(C))). - Dinur $\nearrow$ Gödel prize

Problem 1 - turning CNFs of width $w \to$ width 3

Problem 2 -

Problem 3 - giving PCPP system

r-query turns PCPP reductions into 3-query PCPP reductions

Problem 2

every function on n-bits has DNF width n, size 2^n.

Problem i - Run i

Coding Theory

$f: \mathbb{F}_2^k \to \mathbb{F}_2^n \quad n > k$

s.t. $\text{dist}(f(x), f(y))$ large for all pairs $x, y \quad x \neq y$

(send a message through a noisy channel)

Hamming Code

$n = 2^k$

$x \in \mathbb{F}_2^k$, $f(x) = (0, x_1, x_2, \ldots, x_k, x_1+x_2, x_1+x_3, \ldots, x_{k-1}+x_k, x_1+x_2+x_3, \ldots, x_1+x_2+\cdots+x_n)$

$\text{dist}(f(x), f(y)) = \frac{1}{2}$ if $x \neq y$.