

Boolean Functions

1. Summary

- Dictator Testing
- FKN Thm.

2. Q+A

3. Problems

Def. r -query function tester

choose r queries a priori

Def. Local tester class $\mathcal{L}$

if $f \in \mathcal{L}$, always accepts

rejects w/ probability $\geq \lambda \cdot \text{dist}(f, \mathcal{L})$

Ex. BLR, 3 query test, linear functions.

Dictator testing - Amos theorem

NAE test - pick x, y, z uniformly s.t.

$\text{NAE}(x_i, y_i, z_i) = 1$ for all i

accept if $\text{NAE}(f(x), f(y), f(z))$

Probability of acceptance = $\frac{3}{4} - \frac{3}{4} \text{Stab}_3[f]$

$\text{Prob}[\text{acc}] \geq 1 - \epsilon \rightarrow W[f] \geq 1 - 4\epsilon \rightarrow \text{dictator or anti-dictator (FKN)}$

Avoid anti-dictator, combine BLR + NAE

Can combine r_1 -query, r_2 -query, ..., r_r -query into $\max(r_1, \dots, r_r)$ -query

Lemma. $f: \{-1, 1\}^n \rightarrow \mathbb{R}$, $\deg \leq k$, non-constant
 $\mu = E[f]$, $\sigma = \sqrt{\text{Var}[f]}$

$$\Pr[|f(x) - \mu| \geq \frac{1}{2}\sigma] \geq \frac{1}{16} n^{1-k}$$

FKN: $f: \{-1, 1\}^n \rightarrow \{-1, 1\}$, $W[f] = 1 - \delta$,
then f is $O(\delta)$ -close to χ_i

Proof. $L = f^2$, $\text{Var}[L^2]$ is small, other $W[f] < 1 - \delta$.

Problem 1 - manipulate rejection probability

Problem 2 - testing class of $\{9 \cdot 13^n\}$ w/ an odd

Problem 3 - Why no 2-query # of 15 for dictator

Room i - Problem i