February 11

Thursday, February 11, 2021 1:38 PM

# Boolean Functions

1. Summary
   - LTFs
   - Central Limit Theorem
2. Q+A
3. Problems

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LTF's (linear): $f: \{-1,1\}^n \to \{-1,1\}$

$f(x) = \mathrm{sgn}(\underbrace{a_0 + a_1x_1 + a_2x_2 + \cdots + a_nx_n}_{\text{never } 0})$

PTF's $f(x) = \mathrm{sgn}(p(x))$

PTF's $\downarrow$ polynomial threshold function; $p(x)$ $\downarrow$ polynomial

Thm. If $f, g: \{-1,1\}^n \to \{-1,1\}$, $f$ is LTF, $\hat{f}(S) = \hat{g}(S)$ when $|S| \le 1$, $f = g$

Thm. If $f$ is an LTF, $W^{\le 1}[f] \ge \frac{1}{2}$

Conj. $\ge \frac{2}{\pi}$

Proof. Khintchine-Kahane

$\|\ell\|_1 \ge \frac{1}{\sqrt{2}} \|\ell\|_2$ $\qquad \|\ell\|_p = \mathbb{E}[|\ell(x)|^p]$

$\ell$ $\downarrow$ linear function

$\ell(x) = a_0 + a_1x_1 + \cdots + a_nx_n$

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CLT $a \in \mathbb{R}^n$, $x$ uniform over $\{-1,1\}^n$

$\langle a, x\rangle \sim N(0, \|a\|_2)$

$\langle a, x\rangle \downarrow S$; $N(0,\|a\|_2) \downarrow Z$; $\|a\|_2$ ↳ uniform weight, $\sqrt{\sum a_i^2}$

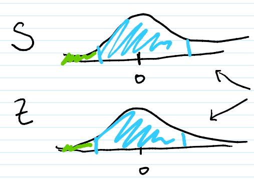
$\frac{1}{2}$ $|\Pr[S \le u] - \Pr[Z \le u]| \le \frac{\max(|a_i|)}{\|a\|_2}$

Berry-Esseen

$\downarrow$

Proof - Fourier Analysis

Real functions

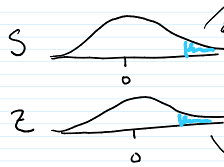


similar Berry-Esseen

bounded third-moment

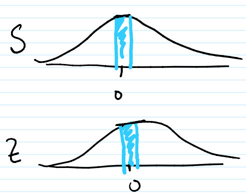
green-regions small $e^{-\|a\|_2^2 \cdot c_1}$

decay similar



Chernoff / Hoeffding $|a_i| \le 1$

$e^{-\|a\|_2^2 \cdot c_2}$



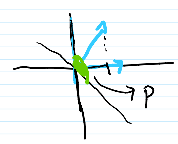
(no spike at 0)

Littlewood-Offord $|a_i| \ge 1$

$\mathrm{Stab}_\rho[\mathrm{Maj}_n] \approx 1 - \frac{2}{\pi} \arccos \rho$

$x, y$ – $\rho$-correlated

$\sum x_i, \sum y_i, \sim$ $\rho$-correlated Gaussians



probability dist. Gaussian depends on distance to origin

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Problem 1 - LTFs Room 1

Problem 2 - LTFs, can assume $a_0 = 0$ Room 2

Problem 3 - $\rho$-correlated bits, Fourier coefficients on singleton sets Room 3

Problem 4 - Fourier transform of random functions Room 4

Problem 1 - LTFs — Room 1

Problem 2 - LTFs, can assume $a_0 = 0$ — Room 2

Problem 3 - p-correlated bits, Fourier coefficients on singleton sets — Room 3

Problem 4 - Fourier transform of random functions — Room 4