February 9

Tuesday, February 9, 2021 1:16 PM

Boolean Functions

1. Summary
   - Switching Lemma
   - LMN Theorem
2. Q+A
3. Problems

Announcements
- Theory slack / seminar (Friday)
- TCS+ February 17 (Circuit Lower Bounds from Pseudorandom Restrictions)

LMN - size s depth-d circuit, spectrum is ϵ-concentrated up to degree $O(\log(\frac{s}{\epsilon})^{d-1} \cdot \log(\frac{1}{\epsilon}))$.

Corollary - poly(n)-size circuit, depth-d, can be learned up to $\frac{1}{\text{poly}(n)}$ error in time $n^{O(\log n)^d}$.

Corollary - If C is a depth-d circuit s.t. $\Pr[C(x) = \chi_{[n]}(x)] \geq \frac{1}{2} + \epsilon_0$, size of C is $2^{\Omega(n^{\frac{1}{d-1}})}$.

↓

poly(n) size, constant depth circuits can't compute parity

→ P vs. NP
can 3-SAT be computed in polynomial time?
↳ PSPACE vs. PH (polynomial-time hierarchy)

Switching Lemma (baby). f - DNF width w, δ - random restriction

$\Pr[f_{J|z} \text{ is not const}] \leq 5\delta w$, $3\delta w$ today's problems

$\Pr[DT(f_{J|z}) \geq k] \leq (5\delta w)^k$

↳ depth of decision tree computing $f_{J|z}$.

Lemma. $\epsilon = \Pr[DT(f_{J|z}) \geq k]$, then (*) f is 3ϵ-concentrated up to degree $3k/\delta$.

Note $\epsilon \geq \Pr[\deg(f_{J|z}) \geq k]$.

Corollary. If f is width w DNF, ϵ-concentrated up to deg $O(w \log \frac{1}{\epsilon})$. → $\hat{\|}f\hat{\|}_2$

Lemma. f is width-w DNF,

$$\sum_{|U| \leq k} |\hat{f}(U)| \leq 2 \cdot (20w)^k \rightarrow \hat{\|}f\hat{\|}_1$$

f is ϵ-concentrated on $\mathcal{F}$, $|\mathcal{F}| \leq w^{O(w \log \frac{1}{\epsilon})}$

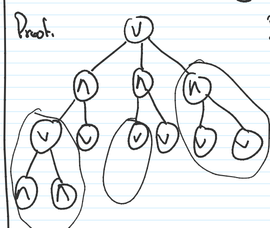
(goal is 2^w)

Thm. f size s, depth d width w circuit (width w → $\log(s/\epsilon)$)

$$\delta = \frac{1}{10w}\left(\frac{1}{10\ell}\right)^{d-2}, \quad \ell = \log\left(\frac{2s}{\epsilon}\right)$$

$$\Pr[DT(f_{J|z}) \geq \log \tfrac{2}{\epsilon}] \leq \epsilon$$

Proof.



Random restrictions in stages

Random restriction on DNF → decision tree → CNF → compress

Random restriction on CNF → decision tree → DNF → compress

$\Pr[\text{random restriction width} \geq w] \leq (\frac{3}{4})^w$ of AND

Problem 1 - Influence of DNF $\leq 2w$, CNF $\leq w$

Problem 2 - $3\delta w$

Problem 1 - Rooms 1, 3

Problem 2 - Rooms 2, 4