February 4

Thursday, February 4, 2021 1:33 PM

Boolean Functions

1. Summary
   1. DNF's
   2. Random Restrictions
2. Q + A
3. Problems

Announcements

- Solutions accessible

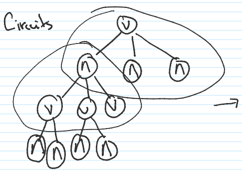
DNF - Formula OR of ANDs

width - max fan-in of AND

size - fan-in of OR

CNF - AND's of ORs - 3-SAT

Circuits



$\rightarrow$ circuits

Nueral Nets

Prop. If f is a width-w DNF,

$I[f] \le 2w$, ϵ-concentrated up to degree $2w/\epsilon$

Can learn in time $n^{O(w)}$

Thm. If f is a size-s DNF, then $\forall \epsilon$,

$\exists g$ computable by a width-$\log(s/\epsilon)$

DNF s.t. $\text{dist}(f, g) \le \epsilon$

Corollary - size s, ϵ-concentrated up to

deg $O(\log(s/\epsilon)/\epsilon)$.

Mansour's conjecture - width-w ϵ-concentrated

on $2^{O(w)}$ coefficients

Mansour $w^{O(w)}$, $s^{O(\log \log s)}$

Thm. $I[f] \le O(\log s)$ f size-s DNF

δ-Random restriction

$\hookrightarrow$ for each input

$+1$ w/ prob $\frac{1-\delta}{2}$

-1 w/ prob $\frac{1-\delta}{2}$

variable w/ prob δ

$\mathbb{E}[\hat{f}_{J|z}(S)] = \delta^{|S|} \cdot \hat{f}(S)$

$\mathbb{E}[\hat{f}_{J|z}(S)^2] = \sum_{U \subseteq [n]} \Pr[U \cap J = S] \cdot \hat{f}(U)^2$

$= \sum_{S \subseteq U} \delta^{|S|} (1-\delta)^{|U \setminus S|} \hat{f}(U)^2$

$\mathbb{E}[I[f_{J|z}]] = \delta I[f]$

DNF - OR of AND's

$\downarrow$

$\frac{1}{2}$ random restriction

for each and

Lemma A is AND, $\frac{1}{2}$-random restriction

$\Pr[A_{J|z} \text{ is width} \ge w] \le \left(\frac{3}{4}\right)^w$ — $e^{-w^2 \cdot C}$

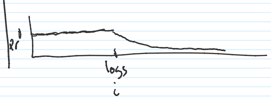
$\Pr[\text{any of the AND's has width} \ge w] \le s\left(\frac{3}{4}\right)^w$ — $s \cdot e^{-w^2 \cdot C}$

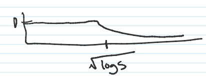
Union-bound

$\mathbb{E}[\text{max width of ANDs}] = \sum_{i=1}^{\infty} \Pr[\text{max width} \ge i]$

$= \sum_{i=1}^{\infty} \min\left(1, s\left(\frac{3}{4}\right)^i\right)$

Generic chaining





$I[f]$ f width w-DNF

$\le 2w$

# of pairs, x, y s.t. $f(x) \ne f(y)$

$x + y$ differ in 1 coordinate

$f(x) = \text{True}$, $f(y) = \text{False}$

True $\rightarrow$ False only 1 of AND's can true

only w ways

Problem 1 - constructing a DNF for any function $\rightarrow$ before 2, 4 — 1, 5

Problem 2 - monotone functions — 2, 6

Problem 3 - $\text{Inf}[f_{J|z}] = \delta \text{Inf}[f]$ w/o Fourier Analysis — 3, 7

Problem 4 - $\chi_{[n]}$, XOR/Parity — 4, 8

$\}$ Rooms

AC_0 - constant depth polynomial-size circuits, Parity $\notin AC_0$