

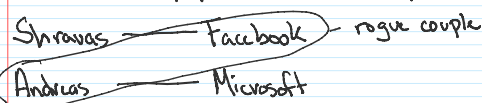
## 1. Stable matching

Stable matching -  $n$  jobs     $n$  applicants  
                                ↓                         ↓  
                            ranked list       ranked list  
                            of preferences    of jobs  
                            of applicants

(stable marriage, applicants men, job women)

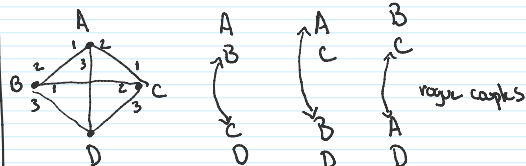


## How to match people in the best way



Def. Proper couple - a job, applicant pair where both job & applicant prefer each other to their match.

Def: Stable matching - a perfect matching <sup>↑</sup> matched  
w/ no rogue couple.



Thm. There exists a stable matching between jobs & applicants

(Gale-Shapely algorithm).

Proof.

1. Each applicant interviews at their favorite job.
2. If a job has multiple applicants, reject all but favorite
3. Applicant gets rejected, crosses job off list
4. Repeat 1-3 until every job has at least one applicant.

Ex

Ex.

Day 1- Only Shrawas interviews at Face book

Day 2 - Andreas also interviews at Facebook  
(rejected Day 1), Facebook prefers  
Andreas, reject Shrawan.

1. Procedure ends
2. Everyone gets matched
3. No rogue copies

1. Everyone has a list of  $n$  jobs. Each time we need to repeat 1-4, someone crossed a job off list. Jobs are never added.
- Therefore <sup>total</sup> # of jobs on all lists is strictly decreasing.

P = for each job  $j$  and applicant  $a$ , if  $j$  is crossed off  $a$ 's list,  $j$  prefers another applicant  $b$  to  $a$ , a  $j$  is  $b$ 's favorite job that's not crossed off.

2. Use contradiction, assume  $a$  didn't get matched, there must be a job  $j$  w/ no match.  $j$  must be crossed off as list, and has an interested applicant by  $P$ , which is a contradiction.

3. Use contradiction, assume  $a \neq j$  are rogue.  
Either  $j$  is crossed off  $a$ 's list, or not.

Case 1      Case 2

Case 1- By  $\mathcal{P}$ ,  $j$  prefers another interested applicant to  $a_j$ ,  $j$  is a can't be rogue.

Case 2-  $a$  prefers their job to  $j$ , as  $j$  can't be rogue.

2. Shrews      1. Facebook  
1. Android      2. Microsoft

Day 1- Shrawans + Andreas interview at Face book  
Face book rejects Shrawans

Day 2 - Shreevas interviews at Microsoft  
Andreas at Facebook