September 28

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Comp Sci 212
1. Sets
2. Sequences
3. Relations

Def. A set is a collection of objects each appearing at most once (multi-set) unordered (sequence).

$[n] = \{1, 2, \ldots, n\}$ $A \subseteq B$ containment (sets)

$a \in B$ element of (element)

| - such that

Ex. $\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\} = \mathbb{Q}$ (rational numbers)

$\{2^k \mid k \in \mathbb{N}\} = \{1, 2, 4, 8, 16, \ldots\}$ - powers of two ($\mathbb{N}$: nonnegative numbers)

$\{a^2 \mid a \in \mathbb{Z}\}$ - set of perfect squares $= \{0, 1, 4, 9, 16, \ldots\}$

$\{a^2 \mid a \in \mathbb{R}, a^2 < 0\}$ - empty set $= \emptyset$

$P(A)$ = set of all subsets of A (power set) $= \{a \mid a \subseteq A\}$

$P([3]) = P(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Ex. $\{s \mid s \subseteq [n], 1 \in s\}$ = set of all subsets of $[n]$ that contain 1.

$\parallel$

$\{s \mid s \in P([n]) \text{ and } 1 \in s\}$

If $n = 3$ $= \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,3\}\}$

$S_r = \{s \mid s \subseteq [n], |s| = r\}$

$E = \{s \mid s \in S_r, 1 \in s\}$

$n = 3$, $r = 2$, $S_r = \{\{1,2\}, \{1,3\}, \{2,3\}\}$

$E = \{\{1,2\}, \{1,3\}\}$

Def. If A is finite, then $|A|$ is the number of elements in A.

$A \subseteq B$ vs. $A \subset B$

$A = B$ ok $A = B$ not okay

Proofs. $A \subseteq B \rightarrow$ if $a \in A$, then $a \in B$

$A = B \rightarrow a \in A$ iff $a \in B$

$A = \emptyset \rightarrow$ for every a, $a \notin A$ (not an element)

contradiction (assume $a \in A$, ... contradiction)

Thm. $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$.

Proof. Use cases. If $a \in A \cup (B \cap C)$, then $a \in A$, or $a \in (B \cap C)$.

Case 1: $a \in A$. Then, $a \in (A \cup B)$ and $a \in (A \cup C)$, so $a \in (A \cup B) \cap (A \cup C)$.

Case 2: $a \in B \cap C$, then $a \in B$, and $a \in (A \cup B)$, Also, $a \in C$, and $a \in (A \cup C)$. Therefore, $a \in (A \cup B) \cap (A \cup C)$.

$A \cup B$ = set of all elements in A or B

$A \cap B$ = set of all elements in A and B

Sequences Ex. $(1,2,3,4) \neq (2,1,4,3)$

$(1,1,1,1) \neq (1,1,1)$

S_i = i^{th} element of sequence s.

$A \times B = \{(a,b) \mid a \in A, b \in B\}$. (set, cartesian product, set) set of all sequences where 1st element is from a, 2nd element is from b

$A_1 \times A_2 \times \cdots \times A_n = \{(a_1, a_2, \ldots, a_n) \mid a_i \in A_i\}$.

If $A_1 = A_2 = \cdots = A_n$, $A_1 \times A \times \cdots \times A_n = A^n$

Ex. $\{0,1\}^n$ = set of all binary strings of length n.

$\{0,1\}^3 = \{(0,0,0), (0,0,1), (0,1,0), (1,0,0), (0,1,1), (1,0,1), (1,1,0), (1,1,1)\}$

Ex. $\{s \mid s \in \{0,1\}^3, s_1 = 1\}$

$= \{(1,0,0), (1,0,1), (1,1,0), (1,1,1)\}$.

set of sequences in $\{0,1\}^3$ w/ first element 1.

Def. A relation is a subset $A \times B$ (domain, range)

Ex. $\{(a,b) \mid a \in \mathbb{R}, b \in \mathbb{R}, b^2 = a\} \subseteq \mathbb{R} \times \mathbb{R}$

(square root). $(4,2)$ $(4,-2)$

$(9,3)$ $(0,0)$.

If every $a \in A$ appears in at most one pair - function $\rightarrow f(a)$ = corresponding element in B.

$(a, f(a)) \in$ relation

function, domain, range: $f: A \rightarrow B$

Inverse image of a function.

$f^{-1}(C) = \{a \mid f(a) \in C\}$ $(C \subseteq B)$

$f^{-1}(b) = \{a \mid f(a) = b\}$ $(b \in B)$

Proofs - Injection, $f(a) = f(b)$ implies $a = b$.

$|f^{-1}(b)| \leq 1$, for all $b \in B$.

Surjection $|f^{-1}(b)| \geq 1$ for all $b \in B$.

For every $a, b \in A$, $f(a) = f(b)$ implies $a = b$.

Ex. $\{(a,b) \mid a \in [n], b \in [2n], b = 2a\}$

$\subseteq [n] \times [2n]$. equivalent

$f: [n] \rightarrow [2n]$, $f(a) = 2a$

f is injective, $f(a) = f(b) \rightarrow 2a = 2b \rightarrow a = b$

f is not surjective, $f^{-1}(b) = \emptyset$ if b is odd.

$f: A \to B$

If every $a \in A$ appears in at most one pair - function $\to$ $f(a)$ = corresponding element in B.

" " at least one pair - total $(a, f(a)) \in$ relation.

If every $b \in B$ appears in at most one pair - injective

" " at least one pair - surjective all four, bijection $\to$ every $a \in A$, $b \in B$, appear in exactly 1 pair.