September 18

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Comp Sci 212

1. Direct Proofs
2. Cases
3. Contrapositive
4. Iff
5. Contradiction

Direct Proof - sequence of implications that combine

Direction matters

(Last class) If $0 \le x \le 2$, then $-x^3+4x+1 \ge 0$

If $0 \le x \le 2$, $\rightarrow$, $\cdots \rightarrow -x^3+4x+1 \ge 0$.

If $-x^3+4x+1 \ge 0$, $\rightarrow \cdots \rightarrow \cdots$

Theorem (Arithmetic Mean-Geometric Mean Inequality)

If $a, b \ge 0$,

$$\frac{a+b}{2} \ge \sqrt{a \cdot b}$$

Ex. $a=2,\ b=2$ $\quad \frac{2+2}{2}=2 \quad \sqrt{2\cdot 2}=2$

$a=1,\ b=9$ $\quad \frac{1+9}{2}=5 \quad \sqrt{1\cdot 9}=3$

(don't include in HW)

Proof. If $a, b \ge 0$, then $(a-b)^2 \ge 0$, or $a^2-2ab+b^2 \ge 0$.

By adding $4ab$ to both sides, we get $a^2+2ab+b^2 \ge 4ab$, or $(a+b)^2 \ge 4ab$.

Therefore, by taking the square root of both sides, because a, b is positive $a+b \ge 2\sqrt{ab}$, or $\frac{a+b}{2} \ge \sqrt{ab}$. $\square$

Cases - If P then Q

If P then P_1 or P_2

If P_1 then Q

If P_2 then Q.

Theorem. If $a+b \ge 16$, either $a \ge 8$ or $b \ge 8$.

$a=17, b=1 \quad a=1, b=17, \quad a=9, b=9$

Proof. Use cases. Either $a \ge 8$ (Case 1), or $a < 8$ (Case 2).

Case 1 - we're done

Case 2 - If $a<8$, then $-a>-8$, and because $a+b \ge 16$, this implies $b \ge 8$. $\square$

Theorem. In every group of 6 people, there are 3 mutual friends, or 3 mutual strangers. (at least is fine) $\rightarrow$ every pair is not friends

Proof. Use cases. Pick person x. x has either at least 3 friends (Case 1), or at most 2 (Case 2).

Case 1. All of x's friends are mutual strangers (Case 1a), or are not $\rightarrow$ Case 1b.

Case 1a. 3 mutual strangers (x has ≥ 3 friends)

Case 1b. y & z are friends w/ each other, and with x. 3 mutual friends.

Case 2. All people x has not met are mutual friends (Case 2a) or not (Case 2b).

Case 2a. There are 3 mutual friends (people x has not met)

Case 2b. y & z are strangers & they're strangers w/ x. 3 mutual strangers. $\square$

Alternate Proof. Use cases, pick person x.

Case 1 - x has exactly 5 friends

Case 2 - x has exactly 4 friends

Case 3 - x has exactly 3 friends

⋮

x has exactly 0 friends.

Contrapositive - If P then Q $\rightarrow$ If not Q then not P.

Theorem. If $a+b \geq 16$, then either $a \geq 8$ or (both) $b \geq 8$.

Proof. Use contrapositive. If $a < 8$ and $b < 8$, then $a+b < 16$. $\square$

Theorem. If $r \geq 0$ and r is irrational, so is $\sqrt{r}$.

irrational: $r \neq \frac{m}{n}$ for an integers m & n.

Proof. Use contrapositive. If $\sqrt{r}$ is rational, then $\sqrt{r} = \frac{m}{n}$ for some integers m & n. Therefore, $r = \frac{m^2}{n^2}$, m^2 & n^2 are both integers, and r is an rational. $\square$

Converse - If Q then P

not always equivalent to If P then Q!

Examples are not proofs.

(counterexamples are proofs)