September 16

Wednesday, September 16, 2020 2:45 PM

Comp Sci 212

1. Administrivia
2. Propositions
3. Direct Proofs

Announcements

- 1st Discussion tomorrow (LaTeX tutorial)

This	Future
Induction	Recursion
Combinatorics	Sorting - merge-sort is the fastest sorting algorithm possible
Probability	Randomized Algorithms
Graph theory	Graph Algorithms
Linear Algebra	Everything

Def. Proposition a statement that is true or false.

Example $2+2=4$ (true)

Example $2+2=5$ (false)

Example

For all $f(x), g(x)$ (differentiable)

$\frac{d}{dx} f(x)g(x) = \left(\frac{d}{dx} f(x)\right)g(x) + f(x)\left(\frac{d}{dx} g(x)\right)$ (product rule).

$= \left(\frac{d}{dx} f(x)\right)\left(\frac{d}{dx} g(x)\right)$ (False)

Example For every non-negative number n, n^2+n+41 is prime. (false)

↳ only divisible by 1 & itself.

n	n^2+n+41
1	43
2	47
3	53 - all prime
⋮	
20	461
⋮	
39	1601
40	$40^2+40+41 = 40(41)+41 = 41^2$ not prime.

Example If a, b, c, d are positive integers $a^4+b^4+c^4$ never equals d^4. (false)

[Euler's conjecture]

$a = 95,800$ $b = 217,519$ $c = 414,560$ $d = 422,481$

(Noam Elkies)

Def. Predicate a proposition that depends on one or more variables.

Example n is a perfect square

$P(n)$ = "n is a perfect square"

$P(4)$, $P(9)$ True $P(5)$, $P(7)$ false

Def. Axiom proposition accepted as being true (does not need to be proved).

This class - use your best judgement

Claim, Lemma, Theorem - types of propositions.

Def. Implication is a statement of the form If P then Q.

Direct Proof. A sequence of implications, each that do not have to be proved, that combine to form its own implication.

"If P_1 then P_2. Therefore, P_3. Therefore P_4"

If P_1 then P_4. ↑ proof.

Theorem. If $0 \le x \le 2$, then $f(x) = -x^3+4x+1 \ge 0$.

Scratch work.

x	$f(x)$
0	1
0.5	2.875
1	4
1.5	3.625
2	1

$-x^3+4x+1 \ge 1$

$-x^3+4x \ge 0$

$-x^3+4x = -x(x-2)(x+2)$ (neg, neg, pos)

Proof. If $0 \le x \le 2$, then $-x \le 0$, $x-2 \le 0$, and $x+2 \ge 0$.

Therefore, $-x(x-2)(x+2) \ge 0$.

Therefore, $-x^3+4x \ge 0$. By adding 1 to both sides, $-x^3+4x+1 \ge 1$.

Therefore, $-x^3+4x+1 \ge 0$. □