Matching Markets

Eg.

eBay : sellers and buyers
adauctions: advertizers and users
uber : drivers and riders

Typically
- one side : long-lived & strategic
- one side : short-lived & behavioral

Setup
- n buyers (strategic), n items (not-strategic)
- buyers want at most one item.
- items can be sold to at most one buyer.
- buyer i has value v_{ij} for item j

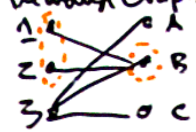
Goal: maximize welfare.

A.k.a. maximum weighted bipartite matching

Example

	Item A	Item B	Item C
Buyer 1	9	8	0
Buyer 2	7	6	2
Buyer 3	6	2	4

Demand Graph



Eg. $p = (5, 1, 3)$

Q: Find optimal matching?

Q: Find market clearing prices.

Matching Algorithms

Alg: Ascending Prices (AP)

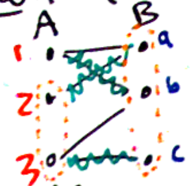
0. prices $p = 0$.
1. construct demand graph D for p
2. if D has perfect matching, output matching & prices, halt.
3. else
   (a) find set S that violates Hall's Thm (marked)
   (b) increase prices of $N(S)$ until some buyer $i \in S$ wants an item $j \notin N(S)$
   (c) repeat (1)

Note: terminates with perfect matching & market clearing prices

Market Clearing

"prices where there is no contention for items, and unsold items have price zero."

Recall: (unweighted) bipartite graphs (A, B, E)



Recall: perfect matching $M \subseteq E$

s.t.
(a) each $v \in A$ is matched } exactly once
(b) each $u \in B$ is matched }

Eg. $M = \{(1,b), (2,a), (3,c)\}$

Recall Hall's Thm: bipartite graph (A, B, E) has perfect matching $\iff \forall S \subseteq A,\ |N(S)| \geq |S|$ (neighborhood)

Def: demand graph D at prices p where $N(i) = \arg\max_j \{v_{ij} - p_j\}$

Def: prices are market clearing if demand graph has perfect matching

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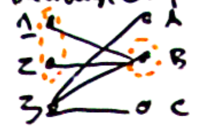

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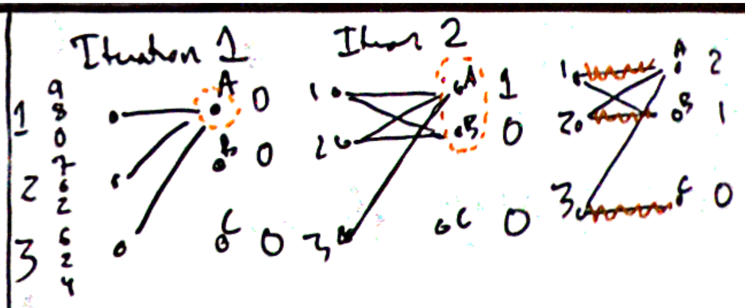

Matching Mechanisms

"if buyers are strategic"

Q: design a mechanism that maximizes welfare in equilibrium?

Mech: externality pricing mechanism

0. solicit bids $\leftarrow \mathbb{B}$
1. compute optimal welfare $W = OPT(\mathbb{B})$ and optimal assignment $\mathbb{X}$
2. compute optimal welfare without i: $W_i = OPT(\mathbb{B}_{-i})$
3. charge bidders externality $p_i = W_i - (W - b_{ij}x_{ij})$
4. output $\mathbb{X}$, output $\mathbb{P}$

A.k.a. "Vickrey-Clarke-Groves (VCG) mechanism"

Thm: truthful bidding is a DSE in Externality Pricing Mechanism.

Cor: EP maximizes welfare in DSE

Proof:

- consider alternate payment $p_i' = -(W - b_{ij}x_{ij})$
- truthtelling utility ($v_{ij} = b_{ij}$) $v_{ij}x_{ij} - (-(W - b_{ij}x_{ij})) = W$.
- but EP maximizes welfare. $\Rightarrow$ optimal to bid truthfully.
- Note: W_i is a constant w.r.t. i's bid. (adding constant to utilities of all bids doesn't change optimal bid)

$\Rightarrow$ with payments $p_i = W_i + p_i'$ optimal to bid truthfully.