CS 396: Online Markets

Lecture 15: Inference for Learning Bidders

Last Time:

- optimization of truthful auctions (cont).
- optimal first-price auctions.
- learning to price.
- learning to auction.

Today:

- value inference (econometrics)
- inference for learning bidders

Exercise: Auction Forensics

Recall:

- two bidders, $U[0,1]$ values, first-price auction
- Bayes-Nash equilibrium strategy: $\sigma(\mathsf{v}) = \mathsf{v}/2$

Setup:

- two bidders, $U[0,1]$ values, first-price auction
- bids are $\mathsf{b}_1 = 0.1$ and $\mathsf{b}_2 = 0.2$

Questions: What are the values

- of bidder 1?
- of bidder 2?

Value Inference

"infer values from bids"

Setup:

- have lots of bid data for current auction
- want to know if new auction is better

Challenge:

- bid data is in equilibrium for current auction.
- how to tell if new auction is good.

Failed Approach: simulate new auction on old bid data.

Example:

- four buyers, uniform values.
- current auction:
  - 2 item first-price auction
  - revenue $= \mathbf{E}\big[2\mathsf{v}_{(3)}\big] = 2 \times 2/5 = 4/5$
- new auction:
  - 3 item first-price auction
  - revenue $= \mathbf{E}\big[3\mathsf{v}_{(4)}\big] = 3 \times 1/5 = 3/5$
- conclusion: 2-item revenue $>$ 3-item revenue
- on bids for 2-item FPA:
  - 2-item revenue = highest 2 bids
  - 3-item revenue = highest 3 bids
  - $\Rightarrow$3-item revenue $>$ 2-item revenue

Gold Standard Approach:

1. infer values from bids
2. calculate new equilibrium and revenue

Main Idea:

- bidders best respond to other bids
- other bids are in the data
- invert the best response function

Example: proportional bids

- two bidder
- proportional bids allocation:

  $\tilde{x}_i(\mathbf{b}) = \mathsf{b}_i / \sum_j \mathsf{b}_j$
- winner pays bid.
- $\mathsf{b}_1 = 1$, $\mathsf{b}_2 = 2$

Q: what is v_1?

Lemma: in winner-pays-bid mechanism with bid-allocation-rule $\tilde{x}(\cdot)$, can infer value as

$$\mathsf{v} = \mathsf{b} + \tilde{x}(\mathsf{b})/\tilde{x}'(\mathsf{b}).$$

Proof:

$\frac{d}{d\mathsf{b}}[(\mathsf{v}-\mathsf{b})\tilde{x}(\mathsf{b})] = (\mathsf{v}-\mathsf{b})\tilde{x}'(\mathsf{b}) - \tilde{x}(\mathsf{b}) = 0 \Rightarrow$
$\mathsf{v} = \mathsf{b} + \tilde{x}(\mathsf{b})/\tilde{x}'(\mathsf{b})$

A: $\mathsf{v}_1 = 1 + 1/3 \times 9/2 = 1 + 3/2 = 5/2$

- $\mathsf{b}_1 = 1$
- $\tilde{x}_1(\mathsf{b}_1) = 1/3$
- $\tilde{x}_1'(\mathsf{b}_1) = 2/9$

Exercise: Proportional Bids

Recall:

- in winner-pays-bid mechanism with bid-allocation-rule $\tilde{x}(\cdot)$, can infer value as $\mathsf{v} = \mathsf{b} + \tilde{x}(\mathsf{b})/\tilde{x}'(\mathsf{b})$.

Setup:

- proportional bids $\tilde{x}_i(\mathbf{b}) = \mathsf{b}_i / \sum_j \mathsf{b}_j$
- winner-pays-bid
- observed bids: $\mathsf{b}_1 = 1$, $\mathsf{b}_2 = 2$

Questions: What is the value v_2 of bidder 2?

Inference for Learning Bidders

Model:

- bidders with static values:
  $\mathbf{v} = (\mathsf{v}_1, \ldots, \mathsf{v}_m)$
- mechanism in round i:
  - bid allocation rule $\tilde{\boldsymbol{x}}^i$
  - bid payment rule $\tilde{\boldsymbol{p}}^i$
- assumption: bidders have low regret

Recall: bidder j has ϵ_j regret for bids $\mathbf{b}^0, \ldots, \mathbf{b}^n$ and alternative bid z:

$$\frac{1}{n}\sum_i \left[\mathsf{v}_j \tilde{x}_j^i(\mathbf{b}^i) - \tilde{p}_j(\mathbf{b}^i)\right]$$
$$\geq$$
$$\frac{1}{n}\left[\mathsf{v}_j \tilde{x}_j^i(\mathsf{z}, \mathbf{b}_{-j}^i) - \tilde{p}_j(\mathsf{z}, \mathbf{b}_{-j}^i)\right] - \epsilon_j$$

Def: $(\mathsf{v}_j, \epsilon_j)$ is **rationalizable** if $\mathbf{b}^0, \ldots, \mathbf{b}^n$ satisfies ϵ_j regret for value v_j.

Goal: identify rationalizable set.

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Changing to bid z:

- $\Delta\tilde{x}_j(\mathsf{z}) = \frac{1}{n}\sum_i \left[\tilde{x}_j^i(\mathbf{b}^i) - \tilde{x}_j^i(\mathsf{z}, \mathbf{b}_{-j}^i)\right]$
- $\Delta\tilde{p}_j(\mathsf{z}) = \frac{1}{n}\sum_i \left[\tilde{p}_j^i(\mathbf{b}^i) - \tilde{p}_j^i(\mathsf{z}, \mathbf{b}_{-j}^i)\right]$

Rearranging: ϵ_j regret $\Rightarrow$ forall z:

$\mathsf{v}_j \Delta\tilde{x}_j(\mathsf{z}) - \Delta\tilde{p}_j(\mathsf{z}) \geq -\epsilon_j$

Conclusion: each z gives a linear constraint on

$(\mathsf{v}_j, \epsilon_j) \in R_j$

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Note: the rationalizable set is convex.

Approach:

- true $(\mathsf{v}_j, \epsilon_j)$ is in rationalizable set.
- reasoanble approach to identify one:
  - v_j that minimizes ϵ_j