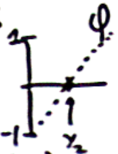
Revenue Maximization

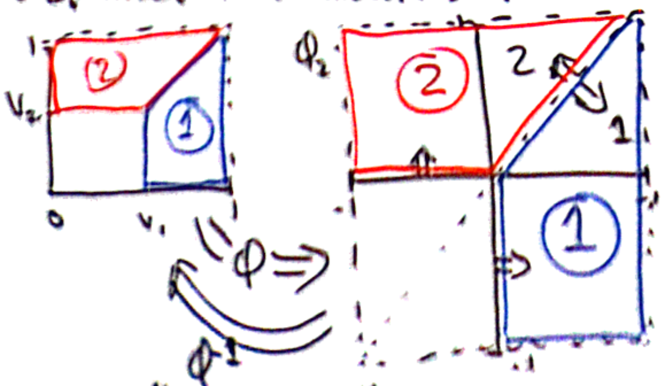
Recall: virtual value: $\varphi(v) = v - \frac{1-F(v)}{f(v)}$

Thm: in multi-bidder mechanisms
expected revenue = expected virtual welfare

Example 1: symmetric bidders



- two bidders, uniform values
- $\varphi(v) = 2v - 1$
- 1 wins if $v_1 > \max(v_2, \varphi^{-1}(0))$
- optimal mechanism: SPA w. reserve $\varphi^{-1}(0) = 1/2$

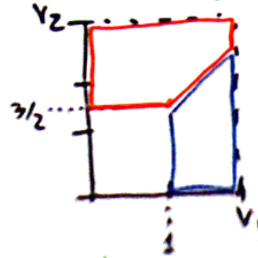


Cor: iid bidders, SPA with reserve is revenue optimal.*

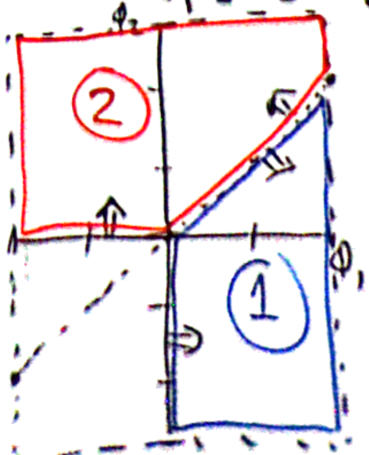
Example 2: asymmetric bidders

- bidder 1:
  - value $U[0,2]$ $\quad F_1(v) = v/2;\ f_1(v) = 1/2$
  - virtual value $\varphi_1(v) = v - \frac{1-v/2}{1/2} = v-(2-v) = \boxed{2v-2}$
- bidder 2: $\quad F_2(v) = v/3;\ f_2(v) = 1/3$
  - value $U[0,3]$
  - virtual value $\varphi_2(v) = v - \frac{1-v/3}{1/3} = v-(3-v) = \boxed{2v-3}$
- bidder 1 wins
  - $\varphi_1(v) \geq 0 \Rightarrow v \geq 1.$
  - $\varphi_1(v_1) \geq \varphi_2(v_2) \Rightarrow 2v_1 - 2 \geq 2v_2 - 3$

  $v_1 \geq v_2 - 1/2.$



optimal mechanism
- reserves $\hat{v}_1 = 1, \hat{v}_2 = 3/2$
- penalty of 1/2 for bidder 2
- SPA-like (charge winner critical value)

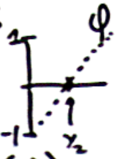


Revenue Maximization

Recall: virtual value: $\varphi(v) = v - \frac{1-F(v)}{f(v)}$

Thm: in multi-bidder mechanisms expected revenue $=$ expected virtual welfare

Example 1: symmetric bidders



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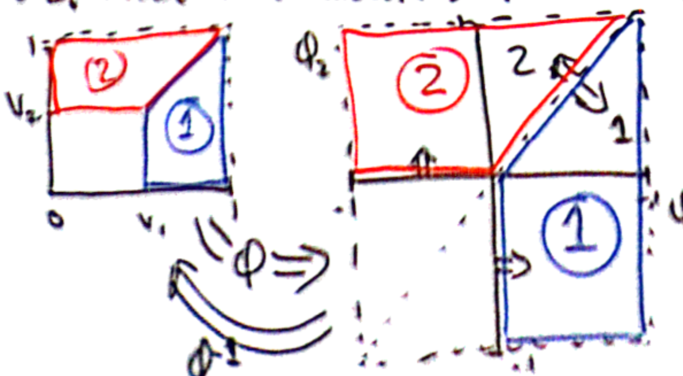

Cor: iid bidders, SPA with reserve is revenue optimal*.

Revenue Optimal First Price Auctions

"truthful auctions are often impractical"

Approach: find first-price auction with same equilibrium allocation rule as optimal truthful mechanism.

Example:

- two bidders, uniform values.

Q: what is revenue-optimal first price auction?

A: first-price auction w. reserve $1/2$.

Q: what is its equilibrium?

- if $v < 1/2 \Rightarrow$ $\boxed{\text{don't bid}}$
  $x(v) = 0$ if $v \le 1/2$.
- if $v > 1/2 \Rightarrow$ symmetric strategy,
  higher value $\Rightarrow$ higher bid
  $\Rightarrow$ $\boxed{\text{higher value wins.}}$

A: same outcome as SPA w. reserve $1/2$.

$\Longrightarrow$ same expected revenue.

Learning to Price

"selling online to one buyer at a time"

Model:

- in round i:
  - buyer i arrives
  - offer a price
  - learn whether buyer takes or leaves price.
- goal: maximize revenue.

Approach: "learn to price" similar to "learning to bid" (partial feedback)

Learning to Auction

"multiple buyers at a time"

Model: symmetric buyers

- assumption: "values are iid"
- in round i:
  - round i: buyers arrive
  - run truthful auction
  - learn values & revenue.
- goal: maximize revenue

Approach: "learning to reserve price" similar to "learning to price" (full feedback)

Model: asymmetric values

- round i:
  - buyers arrive
  - run truthful auction
  - learn values & revenue
- goal: maximize revenue.

Observation: optimal auction only needs order of virtual values to run.

Approach:

- m bidders per round
- ℓ values per bidder (discretized)
- outcome decided by order of values and $\emptyset$.

e.g. "$a_1\ b_1\ a_2\ a_3\ b_2\ \emptyset\ b_3$"

- sell to first bidder in order
- $k \leq m^{m\ell}$
- use online learner with actions = orderings = auctions.
- regret for EW: $2h\sqrt{\frac{\ln k}{n}}$
- auction learning regret: $2h\sqrt{\frac{m\ell \ln m}{n}}$

- in round i:
  - round: buyers arrive
  - run truthful auction
  - learn values & revenue.
- goal: maximize revenue

Approach: "learning to reserve price"

similar to "learning to price" (full feedback)

Model: asymmetric values

- round i:
  - buyers arrive
  - run truthful auction
  - learn values & revenue
- goal: maximize revenue.

Observation: optimal auction only needs order of virtual values to run.