CS 396: Online Markets

Lecture 14: Revenue Maximization and Learning

Last Time:

- revenue of auctions (cont).
- virtual values.
- truthfulness and the revelation principle.
- optimization of truthful auctions.

Today:

- optimization of truthful auctions (cont).
- optimal first-price auctions.
- learning to price.
- learning to auction.

Exercise: Expected Payment

Recall:

- allocation rule:
  $x(v) = \mathbf{Pr}[\text{bidder wins with value } v]$
- can view $x(\cdot)$ as cumulative distribution function of random price.

Setup:

- allocation rule $x(v) = v$

Questions:

- what is expected price offered to the bidder?
- what is expected payment of bidder with value $v = 1/2$?

Revenue Maximization

Recall: virtual value $\varphi(\mathsf{v}) = \mathsf{v} - \frac{1-F(\mathsf{v})}{f(\mathsf{v})}$.

Thm: in multi-bidder mechanisms, expected revenue equals expected virtual welfare.

Example 1: symmetric buyers

- two bidders, uniform values
- $\varphi(\mathsf{v}) = 2\mathsf{v} - 1$
- 1 wins if $\mathsf{v}_1 \geq \max(\mathsf{v}_2, \varphi^{-1}(0))$
- optimal auction:
  second price auction with reserve $\varphi^{-1}(0) = 1/2$

Cor: for i.i.d. buyers second-price auction with reserve $\varphi^{-1}(0)$ is revenue optimal

Example 2: asymmetric buyers

- bidder 1:
  - value: $U[0, 2]$
  - virtual value: $\varphi_1(\mathsf{v}_1) = 2\mathsf{v}_1 - 2$
- bidder 2:
  - value: $U[0, 3]$
  - virtual value: $\varphi_1(\mathsf{v}_1) = 2\mathsf{v}_1 - 3$
- bidder 1 wins
  - when $2\mathsf{v}_1 - 2 \geq 0 \Rightarrow \mathsf{v}_1 \geq 1$.
  - when $2\mathsf{v}_1 - 2 \geq 2\mathsf{v}_2 - 3 \Rightarrow \mathsf{v}_1 \geq \mathsf{v}_2 - 1/2$.

DRAW PICTURE

Revenue Optimal First-price Auction.

"truthful auctions are often impractical"

Approach: find first-price auction with same allocation rule as optimal truthful auction.

Example:

- two bidders, uniform values

Q: what is revenue optimal first-price auction?

A: first-price auction with reserve 1/2

- winner is bidder with highest value over 1/2
  $\Rightarrow x$ as second-price auction with reserve 1/2
  $\Rightarrow$ optimal expected virtual welfare

Exercise: Selling Introductions

Setup:

- you are selling introductions
- two bidders, values $U[0, 1]$
- your mechanism either
  - (a) introduces bidders to each other
  - (b) does not introduce them
- design a truthful mechanism to maximize your revenue.

Questions: What is outcome (introduce or not) in the revenue optimal mechanism when

- $v_1 = 0.9$ and $v_2 = 0.2$?
- $v_1 = 0.8$ and $v_2 = 0.1$?
- $v_1 = 0.6$ and $v_2 = 0.6$?

Learning to Price

Model:

in round i:

- buyer arrives
- offer price
- learn whether buyer takes-it-or-leaves-it

Approach: "learning to price" similar to "learning to bid".

Learning to Auction

Model: symmetric buyers

- assumption: buyers have i.i.d. values

in round i:

- bidders arrive
- run truthful single-item auction
- learn values

Approach: "learning to reserve price" similar to "learning to price"

Model: asymmetric buyers

in round i:

- bidders arrive
- run truthful single-item auction
- learn values

Observation: virtual values only decide order.

Approach:

- m bidders
- ℓ values (discritized)
- virtual welfare optimizing auction is defined by an interleaving of these values and $\emptyset$
  e.g., $a_1, b_1, a_2, a_3, b_2, \emptyset, b_3$
- sell to agent ranked first according to interleaved order (if before $\emptyset$)
- at most $k \leq m^{m\ell}$ interleavings.
  - interleaving determined by:
  - $m\ell$ positions
  - m possible bidders per position
- use online learning with actions = auctions
- recall: regret for Exponential Weights: $2h\sqrt{\frac{\ln k}{n}}$
- regret for learning auction: $2h\sqrt{\frac{m\ell}{n} \ln m}$