## Exercise 13.1: Allocation Rules

## Recall

- allocation rule: $x(\mathbf{v})=\operatorname{Pr}[$ bidder wins with value $\mathbf{v}]$
- probability of winning: $\mathbf{E}_{\mathrm{v} \sim F}[x(\mathrm{v})]$
- expected welfare: $\mathbf{E}_{\mathbf{v} \sim F}[\mathbf{v} x(\mathrm{v})]$


## Exercise 13.1: Allocation Rules

## Setup:

- bidder's value is $v \sim U[0,1]$
- allocation rules for mechanisms $A$ and $B$

$$
x_{A}(\mathrm{v})=\mathrm{v} \quad x_{B}(\mathrm{v})= \begin{cases}1 & \text { if } \mathrm{v}>1 / 2 \\ 0 & \text { otherwise }\end{cases}
$$

## Questions:

- In mechanism A, what is probability of winning? expected welfare?
- In mechanism $B$, what is probability of winning? expected welfare?


## Lecture 13: Revenue Maximization (Cont.)

## Course work:

- Peer Review 3 due tonight (Monday).
- Quiz 1, Weeks 1-3, released tonight, due Wednesday night.
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## Last Time:

- second-price auction with reserve
- revenue of auctions


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## Last Time:

- second-price auction with reserve
- revenue of auctions


## Today:

- revenue of auctions (cont).
- virtual values.
- truthfulness and the revelation principle.
- optimization of truthful auctions.
- optimal first-price auctions.


## Exercise 13.2: Optimal Pricing, Redux

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## Setup:

- you have one item to sell.
- you have value 1 for keeping the item.
- buyer with value from exponential distribution

$$
F(z)=1-e^{-z} \quad f(z)=e^{-z}
$$

## Questions:

- what price should you offer to maximize your expected utility (your revenue if you sell, or your value if you keep the item)?

