Exercise 6.1: Exponential Random Variables

Probability Review

- exponential random variable X with rate λ.
- cumulative distirbution function $F_X(z) = \mathbf{P}r[X < z] = 1 - e^{-\lambda z}$.
- density function $f_X(z) = \frac{d}{dz} F_X(z) = \lambda\, e^{-\lambda z}$.
- expectation $\mathbf{E}[X] = \int_0^\infty z\, f_X(z)\, dz = 1/\lambda$.
- conditional expectation $\mathbf{E}[X \mid X > z] = \frac{1}{1-F_X(z)} \int_z^\infty z\, f_X(z)\, dz$.

Exercise 6.1: Exponential Random Variables

Setup:

- X is exponential distribution with rate $\lambda = 1$.

Question: Evaluate

- its expectation $\mathbf{E}[X]$?
- its conditional expectation $\mathbf{E}[X \mid X > 1]$?
- its conditional expectation $\mathbf{E}[X \mid X > 100]$? (Answer on Canvas)

Lecture 6: Online Learning (Cont)

Last Time:

- online learning (cont)
- warm up: be the leader
- follow the perturbed leader

Lecture 6: Online Learning (Cont)

Last Time:

- online learning (cont)
- warm up: be the leader
- follow the perturbed leader

Today:

- online learning (cont)
- warmup: geometric random variables
- follow the perturbed leader (analysis)

Exercise 6.2: FTPL

Exercise 6.2: FTPL

Setup:

- a follow the perturbed leader algorithm (FTPL, $\epsilon = 1/2$, $h = 1$)
  1. let $V_j^i = \sum_{r=1}^{i} v_j^r$
  2. hallucinate: $v_j^0 \sim$ "geometric with rate $1/2$"
  3. in round i choose $j^i = \text{argmax}_j\, v_j^0 + V_j^{i-1}$
- input:

	1	2	3
Action 1	$1/2$	0	0
Action 2	0	1	1

Question: On this input, what is FTPL probability of action 1

1. in round 1?
2. in round 2? (Answer on Canvas)

   (Hint for round 2: let p be "probability of choosing action i when i is ahead by $1/2$"; write recurrence for p; and solve.)