Exercise 1.3: Uniform Expectation

Probability Review

- The cumulative distribution function (cdf) of random variable X is $F_X(z) = \mathbf{P}r[X < z]$.
- The probability density function (pdf) of X is $f_X(z) = \frac{d}{dz} F_X(z)$.
- The expected value of a random variable is $\mathbf{E}[X] = \int_{-\infty}^{\infty} z\, f_X(z)\, dz$.
- The uniform random variable $X \sim U[0, h]$ has cdf $F_X(z) = z/h$ and pdf $f_X(z) = 1/h$.

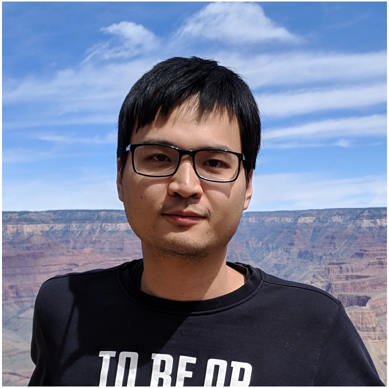
Exercise 1.3: Uniform Expectation

Calculate the expected value $\mathbf{E}[X]$ of uniform random variable $X \sim U[0, 1]$

(Answer on Canvas)

Teaching Assistants

Yiding Feng



Yingkai Li

Course Philosophy

- online markets combines:
  - online algorithm design
  - mechanism design
- lectures:
  - algorithms/mechanisms and proofs.
  - goal: learn why markets work
- projects:
  - hands on experience.
  - goal: learn how theory applies.

Course Syllabus

- Syllabus on Canvas.
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- Lecture videos (posted on Piazza)
- Office hours: TBA.
- Grading is Pass / Not Pass (university guidelines)
  - Lecture exercises (turn in on Canvas as "quizzes")
  - Projects, Project Reports (work in pairs)
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  - Weekly quizzes.
- TBA: Midterm, Final.
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Last Time:

- example: ride sharing
- paradigms: algorithm design, online algorithms, mechanism design
- first-price auction, ascending auction, second-price auction

Lecture 2: Online Allocation

Last Time:

- example: ride sharing
- paradigms: algorithm design, online algorithms, mechanism design
- first-price auction, ascending auction, second-price auction

Today:

- second-price auction (cont)
- online allocation
- prophet inequality
- sequential pricing

Exercise 1.4: Second-price Performance

Probability Review

- *kth order statistic* is kth largest value.
- order statistics denoted: $v_{(1)} \geq \cdots \geq v_{(n)}$
- expected order statistics of uniform variables evenly divide interval
- E.g., for $n = 2$, $v_1, v_2 \sim U[0,1]$, $\mathbf{E}[v_{(1)}] = 2/3$ and $\mathbf{E}[v_{(2)}] = 1/3$.

Exercise 1.4: Second-price Performance

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Exercise 1.4: Second-price Performance

Setup:

- single-item second-price auction
- two buyers, values $v_1, v_2 \sim U[0, 3]$

Question: Calculate expected revenue of seller and expected social welfare (i.e., expected value of winner). (Answer on Canvas)

Exercise 1.5: Two-day Gamble

Exercise 1.5: Two-day Gamble

Setup:

- $n = 2$ prizes
- uniformly distributed $F_1 = F_2 = U[0, 8]$
- realize first prize, claim it, or
- discard it and realize and claim second prize

Question: Find optimal strategy. What is its expected payoff? What is its probability of claiming first price? (Answer on Canvas)