**Mech 3:** second price auction

1) solicit bids
2) winner is highest bidder
3) charge winner <u>second-highest bid</u>

**Thm:** bid = value is a <u>dominant strategy</u> in the second price auction.

**Proof:** consider bidder $i$:

- $\hat{v}_i = \max_{j \neq i} b_j$
- if $b_i > \hat{v}_i$:
  - $i$ wins and pay $\hat{v}_i$
  - utility: $u_i = v_i - \hat{v}_i$

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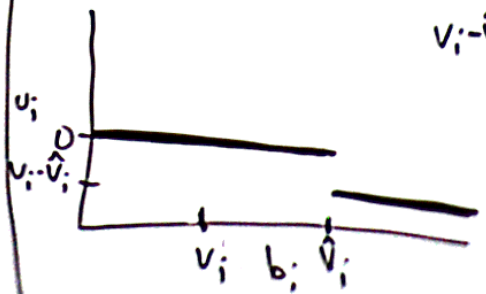
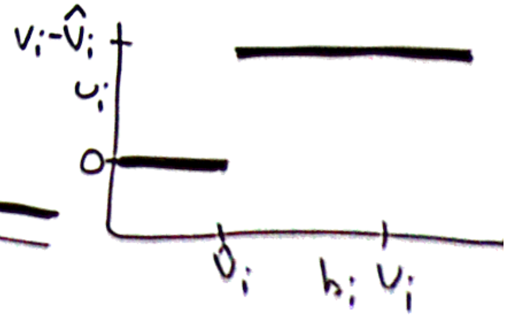
- if $b_i < \hat{v}_i$:
  - $i$ loses
  - utility: $u_i = 0$.
- two cases:

$v_i < \hat{v}_i$ $\qquad\qquad$ $v_i > \hat{v}_i$

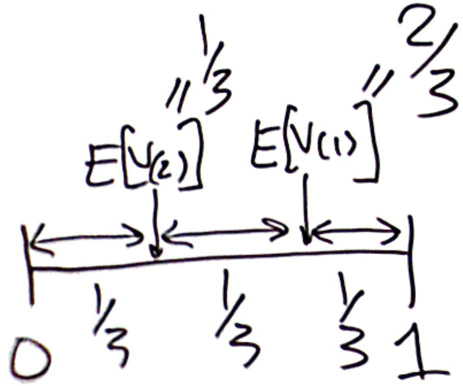



- both cases $b_i = v_i$ is good.
- don't need to know $\hat{v}_i$!

$\Rightarrow$ a dominant strategy.

**Cor:** SPA maximizes social welfare (winner is bidder w. highest value)




$V_1, V_2 \sim U[0,1]$

## Online Algorithms

"buyers show up one at a time must make decision for each buyer before next buyer arrives."

**Note**: no good algorithm without additional information (assumption)

**Example**:

- case 1: $v = (1, 0, \ldots, 0)$
- case 2: $v = (1, 0, \ldots, 100)$

**Assumption**: values are drawn from a known probability distribution

"$v_1 \sim F_1$, $v_2 \sim F_2$, ... $v_n \sim F_n$"

**Example**: $F_1 = F_2 = \cdots = F_n = U[0,1]$

## Online Gambling

- online gambler faces $n$ games.
- game $i$ has prize $v_i \sim F_i$
- gambler plays games in order:
  - realize prize $v_i \sim F_i$
  - either keep prize and quit
  - or discard prize and continue.

Q: what is gambler's optimal strategy

A: "backwards induction"

- on day $n$: take prize!

  $\Rightarrow E[v_n] = \hat{v}_{n-1}$

- on day $n-1$: take prize if $v_{n-1} > \hat{v}_{n-1}$

  $\Rightarrow E[\max(v_{n-1}, \hat{v}_{n-1})]$

- on day $n-2$: take prize if $v_{n-2} \geq \hat{v}_{n-2}$
- ....

$\Rightarrow \hat{v}_1 \geq \dots \hat{v}_{n-1} \geq \hat{v}_n = 0$

$\Rightarrow$ "decreasing sequence of thresholds"

on day $n-1$:

$\Rightarrow E[\max(v_{n-1}, \hat{v}_{n-1})] = \hat{v}_{n-2}$

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# Online Mechanisms

"buyers arrive online
&
buyers are strategic"

"the eBay 'buy it now' problem"

<u>Idea</u>: use thresholds as prices.

<u>Thm</u>: the <u>sequential pricing</u> with prices equal to thresholds $\hat{v}_1, \ldots \hat{v}_n$ from gamblers problem (backwards induction) maximizes social welfare.