June 5

Friday, June 5, 2020 12:03 PM

Comp Sci 212
1. Stable Matching

Announcements
- Final June 11 (info on canvas)

Stable matching - jobs applicants (equal #)

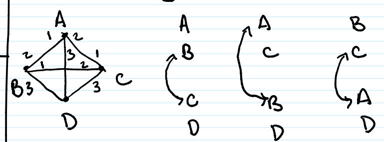
jobs → ranked preferences; applicants → ranked preferences

Def. Rogue couple - a job applicant pair where both job & applicant prefer each other over their current assignment

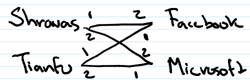
Def. Stable matching - a perfect matching w/ no rogue couples.

Thm. There always exists a stable matching. (bi-partite case)

(Non-bipartite graphs).



Ex.



Shravas — Facebook

Tianfu — Microsoft

Shravas — Microsoft

Tianfu — Facebook

Stable matching.

Proof. (Gale-Shapely algorithm)

1. Each applicant interviews at their favorite job.
2. If job has too many applicants, reject all but their favorite.
3. If an applicant gets rejected, remove the job from their list.
4. Repeat 1-3 until every job has at most one applicant.

Possibility - Day 1, Shravas interview at Facebook
Day 2, Shravas and Tianfu interview at Facebook (Tianfu got rejected on day 1 from his favorite), Facebook rejects Shravas.

1. Procedure ends
2. Every applicant gets a job
3. No rogue couples.

1. Every applicant has a list of jobs, and every time 4 doesn't happen, at least one applicant will remove one job from their list.
Jobs are never added to lists, so total # of jobs on all lists must be strictly decreasing, so can't go on forever.

P = for each job j and applicant a, if j is crossed off a's list, j prefers another applicant b to a, and j is b's favorite (not crossed off).

2. Assume that a does not get a job.
There must a job j that is unfilled.
j rejected a, which by P implies that j has an interested applicant, contradicts j is unfilled.

3. Assume job j and applicant a are rogue.
Either j is crossed off a's list, or not
Case 1: If j is crossed off a's list, j prefers another interested applicant P, so j & a can't be rogue.
Case 2: If j wasn't crossed off a's list, a prefers its job to j, so a & j can't be rogue.
□

Def. matching of a to j is possible if there exists a stable matching matching a & j.

Thm. Each applicant gets optimal possible job.
Proof.
Assume some applicant a eliminates optimal possible job. Assume a eliminated first, let their optimal possible job be j.
Then j prefers another applicant a' to a, a' must prefer j to every possible job (otherwise a wouldn't be the first to cross of optimal).
If a & j are matched, a' & j are a rogue couple, contradicts a & j are a possible match.
□

Thm. Each job gets least optimal possible applicant.

Proof. Assume j is matched with a, but a' is least optimal possible applicant.
By previous theorem j is a's optimal possible job.
If a' and j are matched, then a & j are a rogue couple, so a' is not possible for j.
□

couple, contradicts a & j are a possible match.

$\square$