May 29

Friday, May 29, 2020 2:10 PM

Comp Sci. 212

1. Minimum Spanning Trees

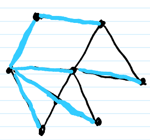
Announcements

- Final June 11 (covers until June 5)
- Practice Final

Def. Spanning tree of a graph $G=(V,E)$

a subgraph $G'=(V,E')$ $(E'\subseteq E)$

s.t. G' to be a tree. $(|E'|=|V|-1)$.



- Blue edges are edges in the spanning tree.

- Minimal connected subgraph w/ all the vertices.

Weighted graphs

$G=(V,E,w)$ $w: E\to\mathbb{R}$

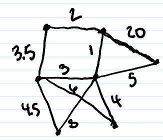
Transportation networks

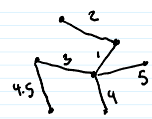
Internet networks

Social networks

weight can represent costs, strength of connection

Problem - How can we find the spanning tree when the sum of edges is minimized? (Minimum Spanning Tree)





Graphs can have multiple MST (if all edges are same weight)

Kruskal's algorithm (assume G is connected)

- Sort the edges
- Consider each edge in increasing order
- For each edge, add it if it doesn't create a cycle.

Thm. Kruskal's algorithm outputs a spanning tree. (T output of Kruskal's algorithm)

Prof. T is acyclic, so just need to prove T is connected. → Every pair of vertices is connected, → path between every pair.

Use contradiction, assume T is not connected, v_1, v_n be not connected in T. Let $(v_1, v_2, \dots, v_n)$ be path in G. Let v_i, v_{i+1} be disconnected in T.

Kruskal's algorithm considers $\{v_i, v_{i+1}\}$, if adding $\{v_i, v_{i+1}\}$ to T creates a cycle, then v_i must already be connected to v_{i+1} in T, which is a contradiction.

(assume weights of edges in G are distinct)

Thm. Kruskal's algorithm outputs a MST.

Proof. Use contradiction, let T be tree output by Kruskal's algorithm. Assume T is not minimal. Let e be first edge added to T not in a MST M.

Adding e to M creates a cycle. The cycle must have one edge f not in T. Let M' be M w/ e but without f. M' is also a spanning tree. Will show that M' has a smaller weight than M, contradict the fact that M is a MST.

total weight of M' = total weight of $M - w(f) + w(e)$

Weight of M' is smaller if $w(f) > w(e)$.

If $w(f) < w(e)$, f is not in T, therefore adding f to T would have created a cycle, edge weights less than $w(e)$.

This implies M contains all the edges in the cycle, M is not a spanning tree contradicts choice of M.