May 20

Wednesday, May 20, 2020 2:51 PM

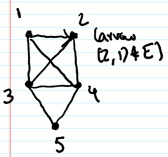
Comp Sci 212

1. Graphs

Graphs

Def. (Graph) Vertex set V, Edge set $E \subseteq V \times V$

$G = (V, E)$



(arrow $(2,1) \notin E$)

$V = \{1, 2, 3, 4, 5\}$

$E = \{(1,2), (1,3), (1,4), (2,3), (2,4), (3,4), (3,5), (4,5), \ldots\}$ (and reverse)

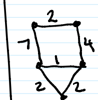
Graphs can represent

- Social networks → V = people E = friendships
- Transportation networks → V = stops E = routes
- Applications → V = union of people & jobs E = applications

Directed if $(v,u) \in E$ does not imply $(u,v) \in E$ (Instagram) (draw edges using arrows)

Undirected, $(v,u) \in E$ implies that $(u,v) \in E$ (Facebook) (draw w/o arrows)

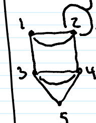
Weighted graphs, weight function $w: E \to \mathbb{R}$



$G = (V, E, w)$

(length of route, strength of friendship)

Multi-graphs - E is a multi-set



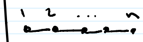
← self-loop

Def. self loop is an edge of the form (v,v)

Def. Simple graph a undirected graph w/ no self-loops, not weighted, not a multi-graph.

$E \subseteq$ set of subsets of V of size 2.

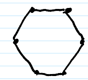
Common graphs $V = [n]$



path

$E = \{\{1,2\}, \{2,3\}, \{3,4\}, \ldots \{n-1, n\}\}$

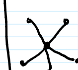
$|E| = n-1$



cycle

$E = \{\{1,2\}, \{2,3\}, \ldots, \{n-1, n\}, \{n, 1\}\}$

$|E| = n$



star

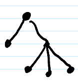
$E = \{\{1,2\}, \{1,3\}, \{1,4\}, \ldots \{1,n\}\}$

$|E| = n-1$

complete graph

E = set of subsets of V of size 2

$|E| = \binom{n}{2}$



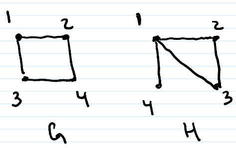
Tree (talk about next class)

Def. (Isomorphism) Let $G = (V_G, E_G)$, $H = (V_H, E_H)$, $f: V_G \to V_H$ is an isomorphism if

- f is a bijection
- If $\{u,v\} \in E_G$ implies that $\{f(u), f(v)\} \in E_H$
- If $\{u,v\} \in E_H$ implies that $\{f^{-1}(u), f^{-1}(v)\} \in E_G$

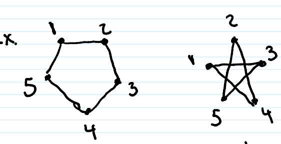
G & H are isomorphic if there exists an isomorphism.

Ex



Both graphs have same # of vertices & edges, but there does not exist an isomorphism.

Ex



$f: [5] \to [5]$ – two graphs are isomorphic

$f(1) = 1$ $f(2) = 3$ $f(3) = 5$ $f(4) = 2$ $f(5) = 4$

Def. u is a neighbor of v if $\{u,v\} \in E$

Def. Degree of v, $\deg(v)$, $d(v)$, d_v is $|\{e \mid e \in E, v \in e\}|$, # of edges that contain v

(in simple graphs = # of neighbors of v)

Thm $\sum_{v \in V} \deg(v) = 2|E|$

(handshake lemma)

Proof. Each edge contributes 1 to the degree of 2 vertices.

(alternative proof)

... $= 1$ if $\{u,v\} \in E$

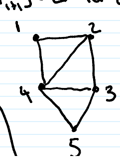
Corollary - In any simple graph, number of vertices w/ odd degree is even.

Proof. Use contradiction, assume the number is odd

The sum of degrees is odd, contradiction □

Paths

Def. A path is a sequence of vertices $(v_1, v_2, \ldots, v_n)$ s.t. $\{v_i, v_{i+1}\} \in E$ for all i (repeats ok)



1-2-4-5

1-4-3

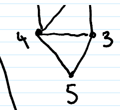
of 2 vertices.

(alternative proof)

Prof. S = set of subsets of size 2, $f(\{v,w\}) = \begin{cases} 1 & \text{if } \{v,w\} \in E \\ 0 & \text{o.w.} \end{cases}$

$$\sum_{v \in V} \deg(v) = \sum_{v \in V} \sum_{\substack{w \in V \\ w \neq v}} f(\{v,w\}) = \sum_{\substack{v,w \in V \\ v \neq w}} f(\{v,w\}) = 2|E|$$

□



1-4-3

Def. u & v are connected if there is a path from u to v.

Def. Graph is connected if every pair of vertices is connected.

Ex. Unconnected graph