April 22

Wednesday, April 22, 2020 2:01 PM

Comp Sci 212

1 Counting Rules

Announcements
- Plan for exams (take one)
- Video on is okay / encouraged

Counting

Bijection rule - If $f: A \to B$ is a bijection, $|A| = |B|$ (every $a \in A$, $b \in B$ appears in exactly 1 pair)

Setting - Donut

3 types of donuts - C, V, S

Buy 8 donuts

How many ways to buy 8?

A = all the ways to buy 8 donuts - set of multi-sets (way of buying donuts)

B = set of strings / sequences w/ 8 0s and 2 1s $B \subseteq \{0,1\}^{10}$

$\{C, C, C, S, S, S, S, S\} \in A$

$(0,0,0,1,1,0,0,0,0,0) \in B$

$f: A \to B$ $f(a) = 0$ # of C's in a, 1, 0 # of Vs in a, 1, 0 # of S's in a

$f(\{C, V, V, V, V, S, S, S\}) = 0100001000$

Thm. f is a bijection

Proof. f is total, function

Let $s = s_1 s_2 s_3 s_4 s_5 s_6 s_7 s_8 s_9 s_{10}$

Let $s_i = 1$, $s_j = 1$, for $i, j \in [10]$ ($= \{1, 2, \dots, 10\}$) rest 0. $i<j$

$f^{-1}(s) = \{a \mid f(a) = s\}$

$= \{\{\underbrace{C, C, \dots, C}_{i-1 \text{ times}}, \underbrace{V, \dots, V}_{j-i-1 \text{ times}}, \underbrace{S, \dots, S}_{10-j \text{ times}}\}\}$

$|f^{-1}(s)| = 1$,

therefore f is a bijection $\square$

Corollary $|A| = |B|$, by bijection rule.

Product rule - $|A \times B| = |A| \cdot |B|$

Ex. 3 course meal

5 appetizers A

10 entrees B

7 desserts C

# of meals $= |A \times B \times C| = |A| \cdot |B| \cdot |C| = 5 \cdot 10 \cdot 7 = 350$.

Thm. # of strings / sequences of 0's & 1's, of length n is 2^n.

Proof. $|\text{Set of sequences}| = |\{0,1\}^n|$

$= |\{0,1\} \times \{0,1\} \times \cdots \times \{0,1\}|$

$= |\{0,1\}| \cdot |\{0,1\}| \cdot \cdots |\{0,1\}|$

$= 2^n$ $\square$

$\{0,1\} \times \{0,1\} = \{(0,0), (0,1), (1,0), (1,1)\}$

Sum rule. If $A \cap B = \emptyset$ (disjoint)

$|A \cup B| = |A| + |B|$

Ex. I have 4 sweaters from H&M A

2 sweaters from Uniqlo B

1 sweater from my dad C

$|A \cup B \cup C| = |A| + |B| + |C| = 7$ sweaters total.

Theorem. There are 49,950 odd numbers between 100 and 100,000.

Proof. $A = \{x \mid x \text{ is odd}, 100 \le x \le 100{,}000\}$.

$A_3 = \{x \mid x \text{ is odd}, 100 \le x < 1000\}$

$A_4 = \{x \mid x \text{ is odd } 1000 \le x < 10000\}$

$A_5 = \{x \mid x \text{ is odd } 10000 \le x < 100000\}$

By sum rule, $|A| = |A_3| + |A_4| + |A_5|$

$D_1 = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_2 = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$

$D_3 = \{1, 3, 5, 7, 9\}$.

$|A_3| = |D_2 \times D_1 \times D_3| = 9 \cdot 10 \cdot 5 = 450$

$|A_4| = |D_2 \times D_1 \times D_1 \times D_3| = 9 \cdot 10 \cdot 10 \cdot 5 = 4500$

$|A_5| = |D_2 \times D_1 \times D_1 \times D_1 \times D_3| = 9 \cdot 10 \cdot 10 \cdot 10 \cdot 5 = 45{,}000$ $\square$

Therefore $|A| = 450 + 4500 + 45000 = 49{,}950$

Setting - Race w/ n runners - set $R = \{\text{Rikki}, \dots, \}$

How many ways are there to assign 1st, 2nd, 3rd?

Try bijection ways $\to R \times R \times R$

But $(\text{Rikki}, \text{Rikki}, \text{Rikki}) \in R \times R \times R$

S = set of ways to assign 1st, 2nd, 3rd

$n_1 = n$, $n_2 = n-1$, $n_3 = n-2$, so $|S| = n(n-1)(n-2)$

Generalized Product Rule

If S is a set of length-k sequences, $(s_1, s_2, \dots, s_k)$ - sequence

- n_1 is # of possibilities for s_1
- n_2 is # of possibilities for s_2 given s_1

⋮

- n_k is # of possibilities for s_k given $s_1, \dots, s_{k-1}$

$|S| = n_1 \cdot n_2 \cdot n_3 \cdots n_k$