April 17

Friday, April 17, 2020 1:59 PM

Comp Sci 212

1. Sets
2. Sequences
3. Relations / Functions

Def. A set is a collection of objects each appearing at most once (multi-set) unordered. (sequence)

- $[n] = \{1, 2, 3, \dots n\}$

$A \subseteq B$ ← sets — containment

$a \in A$ — elmt of (a → element)

$|$ - such that

Ex. (set-builder notation)

$\{ \frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0 \} = \mathbb{Q}$

$\{ 2^k \mid k \in \mathbb{N} \} = \{1, 2, 4, 8, 16, \dots\}$

$\mathbb{N}$ → non-negative integers (includes 0)

$\{ a^2 \mid a \in \mathbb{Z} \}$ - perfect squares

$\{ a^2 \mid a \in \mathbb{R},\ a^2 < 0 \}$ - empty set

$P(A)$ = set of all subsets of A (power set)

$= \{ a \mid a \subseteq A \}$

$P([3]) = P(\{1,2,3\}) = \{ \emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\} \}$

$\{ s \mid s \subseteq [n], 1 \in s \}$ = set of all subsets of $[n]$ that contain 1.

$= \{ s \mid s \in P([n]), 1 \in s \}$

If $n = 3$, $= \{ \{1\}, \{1,2\}, \{1,3\}, \{1,2,3\} \}$

Def. If A is finite, $|A|$ = # of elements in A

$S_r = \{ s \mid s \in P([n]), |s| = r \}$

$E = \{ s \mid s \in S_r, 1 \in s \}$

$\boxed{A \subseteq B}$ vs $A \subset B$

$A = B$ is okay

$A \subset B$ → strict: $A = B$ not okay

Proofs.
$A \subseteq B \rightarrow$ if $a \in A$ then $a \in B$

$B \subseteq A \rightarrow$ if $b \in B$ then $b \in A$

$A = B \rightarrow a \in A$ iff $a \in B$

$A = \emptyset$ (empty set) for every a, $a \notin A$ ($\notin$ → not an element of)

Thm. $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Proof. Use cases, if $a \in A \cup (B \cap C)$ then either $a \in A$, or $a \in B \cap C$

Case 1. If $a \in A$, then $a \in A \cup B$ and $a \in A \cup C$, so $a \in (A \cup B) \cap (A \cup C)$.

Case 2. If $a \in B \cap C$, then $a \in B$, therefore $a \in A \cup B$, and $a \in C$, so $a \in A \cup C$. This implies $a \in (A \cup B) \cap (A \cup C)$

If $n = 3$, $r = 2$

$S_r = \{ \{1,2\}, \{1,3\}, \{2,3\} \}$

$E = \{ \{1,2\}, \{1,3\} \}$

Sequences

Ex. $(1, 2, 3, 4) \neq (2, 1, 4, 3)$

$s = (1, 1, 1, 1)$

s_i = i^{th} element of the sequence (1-index)

$A \times B = \{ (a, b) \mid a \in A, b \in B \}$

× → cartesian product

$A \times B$ → set

(a, b) → sequence (looks like interval) but it's not

$A_1 \times A_2 \times \dots \times A_n = \{ (a_1, a_2, a_3, \dots a_n) \mid a_i \in A_i \}$

If $A = A_1 = A_2 = \dots = A_n$,

$A_1 \times A_2 \times \dots \times A_n = A^n = \underbrace{A \times A \times A \times \dots \times A}_{n \text{ times}}$

Ex $\{0,1\}^n$ = set of all binary strings of length n

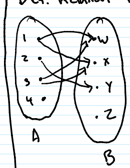
$\{0,1\}^3 = \{ (0,0,0), (0,0,1), (0,1,0), (0,1,1), (1,0,0), (1,0,1), (1,1,0), (1,1,1) \}$

$\{ s \mid s \in \{0,1\}^3, s_1 = 1 \} = \{ (1,0,0), (1,0,1), (1,1,0), (1,1,1) \}$.

□

Def. Relation from A to B is a subset $A \times B$.

A ↑ domain, B ↑ range



$\{ (1,w), (1,x), (2,y), (3,w), (3,x) \}$

If every $a \in A$ appears in at most 1 pair - function

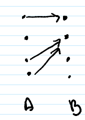
at least 1 pair - total

If every $b \in B$ appears in at most 1 pair - injective

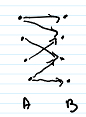
at least 1 pair - surjective

If every $a \in A$, $b \in B$ appears in exactly 1 pair — exactly, exactly - bijection

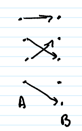
Function



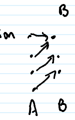
Total



Injection



Surjection



Function, $f: A \to B$, $f(a)$ corresponding element in B.

$(a, f(a))$ in set

Inverse $f^{-1}(b) = \{ a \mid f(a) = b \}$

If f is injective - f^{-1} is a function

If f is surjective - f^{-1} is total

Proofs - f is injective, if $f(a) = f(b) \rightarrow a = b$

f is surjective, $|f^{-1}(b)| \geq 1$

Ex. $\{ (a,b) \mid a \in [n], b \in \{\text{even}, \text{odd}\}, a \text{ is "}b\text{"} \} \in [n] \times \{\text{even}, \text{odd}\}$

$n = 3$, $= \{ (1, \text{odd}), (2, \text{even}), (3, \text{odd}) \}$

(to be continued next class)