April 8

Wednesday, April 8, 2020 2:11 PM

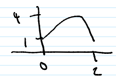
Comp Sci 212

1. Direct Proofs
2. Cases
3. Contrapositive
4. Iff

Announcements
- Office hours schedule is up
- Discussion today (LaTeX tutorial)

Thm. If $0 \le x \le 2$, then $f(x) = -x^3 + 4x + 1 \ge 0$

x	$f(x)$
0	1
0.5	2.875
1	4
1.5	3.625
2	1



$-x^3 + 4x + 1 \ge 1$

$-x^3 + 4x \ge 0$ sames $\underset{\text{neg}}{-x}\ \underset{\text{neg}}{(x-2)}\ \underset{\text{pos}}{(x+2)} \ge 0$

Proof. If $0 \le x \le 2$, then $-x \le 0$, $x - 2 \le 0$, $x + 2 \ge 0$

Therefore $-x(x-2)(x+2) \ge 0$.

Therefore $-x^3 + 4x \ge 0$, or $-x^3 + 4x + 1 \ge 0$ □

important part

$-x^3 + 4x + 1 \ge 1$ and $1 \ge 0$

Thm. (AM-GM) If $a, b \ge 0$

$\frac{a+b}{2} \ge \sqrt{ab}$

Ex. If $a = 2$, $b = 2$, $\frac{a+b}{2} = 2$, $\sqrt{ab} = 2$

$a = 1$, $b = 9$, $\frac{a+b}{2} = 5$ $\sqrt{ab} = 3$

Proof. $(a-b)^2 \ge 0$, or $a^2 - 2ab + b^2 \ge 0$.

By adding $4ab$ to both sides,

$a^2 + 2ab + b^2 \ge 4ab$, i.e. $(a+b)^2 \ge 4ab$.

Because a & b are non-negative, take $\sqrt{\ }$ of both sides, $a + b \ge 2\sqrt{ab}$,

or that $\frac{a+b}{2} \ge \sqrt{ab}$ □

Reverse → If $\frac{a+b}{2} \ge \sqrt{ab}$, then $(a-b)^2 \ge 0$

different statement

Cases If P then Q

↓

If P then P_1 or P_2

If P_1 then Q

If P_2 then Q

Thm. If $a + b \ge 16$, either $a \ge 8$ or $b \ge 8$.

$a = 11$, $b = 1$ $\quad$ $a = 1$, $b = 17$ $\quad$ $a = 9$ $b = 9$

Proof. Use cases. Either $a \ge 8$, or $a \le 8$.

Case 1: If $a \ge 8$, the conclusion is satisfied

Case 2: If $\underline{a \le 8}$, then because $\underline{a + b \ge 16}$,

$b \ge 8$.

Theorem In every group of 6 people, there are either 3 mutual friends, or 3 mutual strangers.

Proof. Use cases, Pick person x. Either x has at least 3 friends, or at most 2 friends.

1 $\qquad$ 2

Case 1: All of x's friends are either mutual strangers, or there are two friends y & z, y & z friends.

1a

1b

Case 1a: At least 3 mutual strangers (x's friends)

Case 1b: x, y, & z are friends.

Case 2: All people x has not met are either all friends, or there are two, y & z, y & z are strangers.

2a

2b

2a: At least 3 mutual friends (people x hasn't met)

2b: x, y, z are mutual strangers

□

Either x has 5 friends – Case 1

4 friends – Case 2

3 friends – Case 3

⋮ $\qquad$ ⋮

0 friends – Case 6

- 1a – all mutual strangers ✓
- 1b – all mutual friends $\quad$ fine
- 1c –

| – succint

Contrapositive If P then Q, equivalent If not Q, then not P

Theorem. If $a + b \ge 16$, then $a \ge 8$ or $b \ge 8$

Proof. Use contrapositive, if $a < 8$ and $b < 8$ then $a + b < 16$ □

Theorem. If $r \ge 0$, and r is irrational, then $\sqrt{r}$ is irrational.

↳ $r \ne \frac{m}{n}$ for integers m, n

Proof. Use contrapositive

If $\sqrt{r}$ is rational, then $\sqrt{r} = \frac{m}{n}$ for integers m, n

Therefore $r = \frac{m^2}{n^2}$, m^2, n^2 are integers, so r is rational.

Converse If Q then P $\ne$ If P then Q