April 6

Monday, April 6, 2020 1:16 PM

Comp Sci 212
1. Administrivia
2. Propositions
3. Direct Proofs

This class	Future
Induction	Recursion
Combinatorics	Lower bounds (can you sort faster than merge-sort?)
Probability	Randomized Algorithms
Graph Theory	Graph Algorithms
	
Number Theory	Cryptography

Def. Proposition a statement that is true or false

Example $2+2=4$ (true)
$2+2=5$ (false)

For all differentiable functions $f(x)$, $g(x)$

$$\frac{d}{dx} f(x)g(x) = \left(\frac{d}{dx} f(x)\right) g(x) + f(x)\left(\frac{d}{dx} g(x)\right)$$

(Product rule)

$$= \left(\frac{d}{dx} f(x)\right)\left(\frac{d}{dx} g(x)\right) \text{ (false)}$$

Ex. For every non-negative integer n, (false)
$P(n)$ = "n^2+n+41 is prime" → number only divisible by 1 & itself

$P(1) = 43$
$P(2) = 47$
$P(3) = 53$
$\vdots$
$P(29) = 461$
– all primes

$P(39) = 1601$
$P(40) = 40^2 + 40 + 41$
$= 40^2$, not prime

Example. If a, b, c, d are positive integers
$a^4 + b^4 + c^4 \neq d^4$

[Euler's conjecture]

$a = 95,800$, $b = 217,519$, $c = 414,560$
$d = 422,481$, then $a^4 + b^4 + c^4 = d^4$
(Noam Elkies)

Def. Predicate a proposition that depends on one or more variables.

Example. n is a perfect square.
$P(n)$ = "n is a perfect square"
$P(4)$, $P(9)$ = True $P(5)$, $P(7)$ = False

Def. Axiom proposition accepted as being true (doesn't need to be proved)
This class – use best judgment

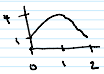
Def. Implication statement of the form "IF P then Q"

Def. Direct Proofs – Sequence of implications, that each do not have to be proved, that combine to form its own implication.

"If P_1 then P_2. Therefore P_3. Therefore P_4."
If P_1 then P_4

Theorem If $0 \leq x \leq 2$, then $f(x) = -x^3 + 4x + 1 \geq 0$.

x	$f(x)$
0	1
0.5	2.875
1	4
1.5	3.625
2	1



Theorem, Lemmas, Claim
types of propositions