March 9

Monday, March 9, 2020 3:37 PM

Comp Sci 212

1. Linear Algebra (not on final)

Announcements

- Final March 17 12pm-2pm, LR2

Linear Algebra

$\mathbb{R}^n$ = set of vectors with n coordinates, real numbers

$v \in \mathbb{R}^n \quad v = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{bmatrix}$

$\mathbb{R}^{n \times m}$ = set of matrices dimension $n \times m$

$A \in \mathbb{R}^{n \times m} \quad A = \begin{bmatrix} A_{11} & A_{12} & \cdots & \\ A_{21} & A_{22} & \cdots & \\ \vdots & & & \\ & & & A_{nm} \end{bmatrix}$

Def. Inner product, $\langle x, y \rangle \quad x, y \in \mathbb{R}^n$

$$\langle x, y \rangle = \sum_{i=1}^{n} x_i y_i$$

$$\langle x, x \rangle = \sum_{i=1}^{n} x^2 = \|x\|_2^2$$

ex. $x = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix} \quad y = \begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

$\langle x, y \rangle = 2 \cdot 2 + 1 \cdot (-1) + 3 \cdot (-1) = 0$

$\langle x, x \rangle = 2 \cdot 2 + 1 \cdot 1 + 3 \cdot 3 = 14 = \|x\|_2^2 \rightarrow \|x\|_2 = \sqrt{14}$

Def. x, y are orthogonal if $\langle x, y \rangle = 0$.

Multiplication

$A \in \mathbb{R}^{n \times m} \quad B \in \mathbb{R}^{m \times p}$, then $\quad B = \begin{bmatrix} b_1 & b_2 & \cdots & b_p \end{bmatrix}$

$AB \in \mathbb{R}^{n \times p}$

$(AB)_{ij} = \sum_{k=1}^{m} A_{ik} B_{kj} \quad A = \begin{bmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \end{bmatrix} \quad \begin{bmatrix} \langle a_1, b_1 \rangle & & \\ & & \\ & & \langle a_n, b_p \rangle \end{bmatrix}$

AB

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 4 & 1 & 7 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 7 \\ 4 & 1 & 7 \\ 2 & 2 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix} \quad \times$$

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 3 \\ 1 & 1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} 5 & 7 \\ 3 & 3 \\ 4 & 5 \end{bmatrix} \leftarrow$$

product

Also works w/ matrix-vector, $v \in \mathbb{R}^n$, also in $\mathbb{R}^{n \times 1}$

$$\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 7 \\ 4 & 1 & 7 \\ 2 & 2 & 2 \end{bmatrix} \begin{bmatrix} 8 \\ 11 \\ 4 \end{bmatrix} \leftarrow$$

product

If $A \in \mathbb{R}^{n \times n}$, A is function

$A: \mathbb{R}^n \rightarrow \mathbb{R}^n$

Def. Identity matrix $I_n \in \mathbb{R}^{n \times n}$

$$(I)_{ij} = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{o.w.} \end{cases}$$

$$\begin{bmatrix} 1 & & 0 \\ & 1 & \\ 0 & & 1 \end{bmatrix}$$

Fact. $AI_n = I_n A = A$ for $A \in \mathbb{R}^{n \times n}$