The following Chernoff bound applies to independent random variables.

**Theorem 1.** *Let $X_1, \ldots, X_n$ be independent random variables such that $\mathbb{E}[X_i] = 0$ and $|X_i| \leq 1$ for all $i$. Then,*

$$\Pr[X_1 + \cdots + X_n \geq u\sqrt{n}] \leq \exp(-u^2/(4e)).$$

*Proof.* The left-hand side can be rewritten as,

$$\Pr[\exp(t(X_1 + \cdots + X_n)) \geq \exp(ut\sqrt{n})] \leq \frac{\mathbb{E}[\exp(t(X_1 + \cdots + X_n))]}{\exp(ut\sqrt{n})}$$

for some positive $t$ to be chosen later. The inequality follows by Markov's inequality. Additionally, we have that

$$\mathbb{E}[\exp(t(X_1 + \cdots + X_n))] = \prod_{i=1}^{n} \mathbb{E}[\exp(tX_i)] \tag{1}$$

by the independence of the $X_i$. We have that

$$\begin{aligned}
\mathbb{E}[\exp(tX_i)] &= \mathbb{E}\left[1 + tX_i + \frac{t^2X_i^2}{2!} + \frac{t^3X_i^3}{3!} + \cdots\right] && \text{(Taylor series)} \\
&= 1 + t\mathbb{E}[X_i] + \frac{t^2\mathbb{E}[X_i^2]}{2!} + \frac{t^3\mathbb{E}[X_i^3]}{3!} + \cdots && \text{(Linearity of expectation)} \\
&= 1 + \frac{t^2\mathbb{E}[X_i^2]}{2!} + \frac{t^3\mathbb{E}[X_i^3]}{3!} + \cdots && (\mathbb{E}[X_i] = 0) \\
&\leq 1 + \frac{t^2}{2!} + \frac{t^3}{3!} + \cdots && (|X_i| \leq 1) \\
&\leq 1 + \frac{t^2}{2!} + \frac{t^2}{3!} + \cdots && (t \leq 1) \\
&\leq \exp(et^2) && \left(\frac{1}{2!} + \frac{1}{3!} + \cdots \leq e, 1 + x \leq \exp(x)\right)
\end{aligned}$$

if $0 \leq t \leq 1$. Thus, we can upper bound Eq. (1) by $\exp(t^2en)$. Thus the upper bound on the probability is $\exp(t^2en - tu\sqrt{n})$ which gives the desired bound by setting $t = u/(2e\sqrt{n})$. $\square$