January 17

Friday, January 17, 2020 3:09 PM

Comp Sci 212

1. Sets
2. Relations / Functions

Announcements

- January 20 Office Hours moved

Def. A set is a collection of objects each appearing at most once (otherwise, multi-set) unordered. (otherwise, sequence)

Ex. $A = \{1, 2, 10\}$
$B = \{\{1, 2\}, 4, \text{yellow}\}$
$C = \{1, 2, 4, 8, \ldots\}$

Common Sets

$\emptyset$ - Empty
$\mathbb{N}$ - Non-negative integers
$\mathbb{Z}$ - Integers
$\mathbb{Q}$ - Rationals
$\mathbb{R}$ - Real numbers
$\mathbb{C}$ - Complex numbers

(a, b) - open range from a to b
$[a, b]$ - closed range
$[n] = \{1, 2, 3, \ldots, n\}$

Relations

$\subset$ - inclusion, strict
$\subseteq$ - inclusion
$\not\subset$, $\not\subseteq$ - not inclusion

$\underbrace{\mathbb{N}}_{\text{subset}} \subset \underbrace{\mathbb{Z}}_{\text{superset}}$
$\mathbb{N} \subseteq \mathbb{Z}$ $\quad \mathbb{Z} \subseteq \mathbb{Z}$
$\mathbb{Z} \not\subseteq \mathbb{N}$

$A \cup B$ - set of elements in A or B (union)
$A \cap B$ - set of elements in A and B (intersection)
$A - B$, $A \setminus B$ - set of elements in A, but not B

$P(A)$ (power set) = set of subsets of A.

$P(\{1,2,3\}) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}\}$

Common Symbols

$\forall$ - for all
$\exists$ - there exists
$\in$ - element of
$\notin$ - not an element of
$\wedge$ - and
$\vee$ - or
$|$ - such that

$1 \in \{1, 2, 4\}$
$\{1\} \subseteq \{1, 2, 4\}$

Ex. $\mathbb{Q} = \{\frac{p}{q} \mid p, q \in \mathbb{Z} \text{ and } q \neq 0\}$
$\{1, 2, 4, 8, \ldots\} = \{2^k \mid k \in \mathbb{N}\}$

$A \times B = \{(a,b) \mid a \in A, b \in B\}$ — sequence, ordered list
$A \times B$ ← Cartesian product

$A \times B \times C = \{(a,b,c) \mid a \in A, b \in B, c \in C\}$ — sequence, ordered list

Def. If A is finite, $|A|$ = # of elements in A.

Ex. $\{a^2 \mid a \in \mathbb{Z}\}$ - set of perfect squares
$\{a^2 \mid a \in \mathbb{Z}, a^2 < 0\}$ - empty

Proofs - $A \subseteq B$ if $a \in A$, then $a \in B$.
$A = B$ $\quad a \in A$ iff $a \in B$
if $a \in A$ then $a \in B$, and if $a \in B$ then $a \in A$
$A = \emptyset$ if for every a, $a \notin A$

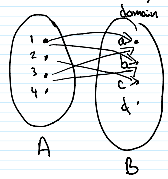
Thm. $A \cup (B \cap C) \subseteq (A \cup B) \cap (A \cup C)$

Proof. Use cases.
If $a \in A \cup (B \cap C)$, either $a \in A$, or $a \in B \cap C$.
Case 1 - If $a \in A$, then $a \in A \cup B$ and $a \in A \cup C$, therefore $a \in (A \cup B) \cap (A \cup C)$.
Case 2 - If $a \in B \cap C$, $a \in B$ and $a \in C$, therefore, $a \in A \cup B$ and $a \in A \cup C$, therefore $a \in (A \cup B) \cap (A \cup C)$ □

Thm. $A = \{x \mid \text{there exists } k \in \mathbb{Z} \text{ s.t. } x = 2k, \text{ and there exist } l \in \mathbb{Z} \text{ s.t. } x^2 = 2l + 1\} = \emptyset$

Proof. Use contradiction.
If $x \in A$ then x is even and therefore x^2 is even. By definition of A, if $x \in A$, then x^2 is odd. □

Def. Relation / Mapping from A to B is a subset of $A \times B$.
A ← domain, B ← range



$\{(1,a), (1,b), (2,c), (3,a), (3,b)\}$

$f: A \to B$ $\quad f(a)$ - element in B, in a pair w/ a

Def. If every $a \in A$ appears at most once in a pair - function
If every $a \in A$ appears at least once, - total
If every $b \in B$ appears at most once - injective / injection
If every $b \in B$ appears at least once, - surjective / surjection

total, function, injective, surjective - bijection

For sets $C \subseteq A$, functions $f: A \to B$
$f(C) = \{f(a) \mid a \in A\}$

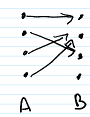
Def. Inverse - pairs $(a,b) \to (b,a)$
$f^{-1}(C) = \{a \mid f(a) \in C\}$, for sets $C \subseteq B$

Facts - If a mapping f is injective, f^{-1} is a function.
If a mapping f is surjective, f^{-1} is total.

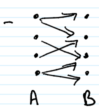
Proofs f is injective, If $f(a) = f(b)$, then $a = b$
f is surjective, $|f^{-1}(\{b\})| \geq 1$ for all $b \in B$.

Ex. $f: \mathbb{R} \to \mathbb{R}$, $f(x) = x^2$
$f^{-1}(x) = \begin{cases} \emptyset & \text{if } x < 0 \\ \{-\sqrt{x}, \sqrt{x}\} & \text{if } x \geq 0 \end{cases}$

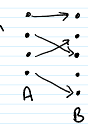
Function -



Total -



Injection



Surjection