January 10

Friday, January 10, 2020 3:05 PM

Comp Sci 212

1. Contradiction
2. Induction

Announcements

Office Hours update/change

Contradiction P is true

If not P then Q

If not P then not Q

Thm. For integers x, x is even iff x^2 is even.

Theorem. $\sqrt{2}$ is irrational.

↳ can't be written as $\frac{m}{n}$ for integers $m + n$

Proof. Use Contradiction. Assume $\sqrt{2}$ is rational. There are integers $m + n$ such that $\sqrt{2} = \frac{m}{n}$, lowest terms, i.e. at most one of $m + n$ is even.

If $\sqrt{2} = \frac{m}{n}$, then $2n^2 = m^2 \to m^2$ is even $\to m$ is even, $m = 2k$ for some integer $k \to m^2 = 4k^2 \to n^2 = 2k^2 \to n^2$ is even $\to n$ is even. □

Theorem. There exist an infinite number of primes.

↳ only divisible by 1 + itself.

Proof. Use Contradiction. Assume there's a finite number of prime numbers. n

The prime numbers are $p_1, p_2, \ldots, p_n$.

Consider $p_1 \cdot p_2 \cdot p_3 \cdots p_n + 1$, not divisible by $p_1, \ldots, p_n$, so must be prime.

Proof that all numbers can be written as a product of primes → Wednesday.

Thm. P is true.

Proof. Assume P is false.

P is true, which contradicts P being false. Therefore P is true.

Induction

Program p that uses recursion - Prove that p works.

Thm. For all $n \in \mathbb{N}$ (for all non-negative integers n),

$$0 + 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}$$

$n=0$ $\quad 0 \quad \frac{0 \cdot 1}{2} = 0$

$n=1$ $\quad 0+1=1 \quad \frac{1 \cdot 2}{2} = 1$

$n=2$ $\quad 0+1+2=3 \quad \frac{2 \cdot 3}{2} = 2$

$n=3$ $\quad 0+1+2+3=6 \quad \frac{3 \cdot 4}{2} = 6$

$n=4$ $\quad 0+1+2+3+4=10 \quad \frac{4 \cdot 5}{2} = 10$

$n=5$ $\quad 0+1+2+3+4+5$

$\frac{4 \cdot 5}{2} + 5 = \frac{4 \cdot 5 + 2 \cdot 5}{2} \quad \frac{5 \cdot 6}{2} = 15$

$n=6$ $\quad 0+1+2+3+4+5+6$

$$\frac{5 \cdot 6}{2} + 6 = \frac{5 \cdot 6 + 2 \cdot 6}{2} = \frac{7 \cdot 6}{2}$$

Idea Use the solution for smaller n, to prove theorem for larger n.

$P(n) = $ "$0 + 1 + \cdots + n = \frac{n(n+1)}{2}$"

Want to prove $P(n)$ is true for all n.

Induction

Prove $P(0)$ - Base case

Prove that for all n, if $P(n)$ then $P(n+1)$ - Inductive step.

Then $P(n)$ is true for all n.

Thm $0 + 1 + \cdots + n = \frac{n(n+1)}{2}$

Proof. Use Induction.

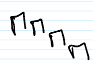
Base case $n=0$, $0 = \frac{0 \cdot 1}{2}$

Inductive step. Assume $0 + 1 + \cdots + n = \frac{n(n+1)}{2}$

$$0 + 1 + \cdots + n + (n+1) = \frac{n(n+1)}{2} + (n+1) = \frac{n(n+1) + 2(n+1)}{2} = \frac{(n+1)(n+2)}{2}$$

□

Dominoes



$P(0) \to P(1) \to P(2) \to P(3) \to \cdots$

Thm. If $P(0)$ is true, and $P(n) \to P(n+1)$ for all non-negative integers n, then $P(n)$ is true for all n. (Induction works).

Proof. Assume there exists n, such that $P(n)$ is false.

Let n be the smallest such non-negative integer. (axiom)

$n \neq 0$, because $P(0)$ is true.

$P(n-1)$ is true, this implies $P(n)$ is true. □

Thm. For all $n \in \mathbb{N}$, r a real number,

$$1 + r + r^2 + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$$

Proof. Use Induction.

Base Case - $n=0$ $\quad 1 = \frac{r-1}{r-1}$

Inductive step - Assume $1 + r + \cdots + r^n = \frac{r^{n+1} - 1}{r - 1}$

$$1 + r + \cdots + r^n + r^{n+1} = \frac{r^{n+1} - 1}{r - 1} + r^{n+1} = \frac{r^{n+1} - 1 + r^{n+2} - r^{n+1}}{r - 1} = \frac{r^{n+2} - 1}{r - 1}$$

□