January 8

Wednesday, January 8, 2020 3:15 PM

Comp Sci 212

1. Cases
2. Contrapositive
3. Iff
4. Contradiction

Announcements

- 1st Discussion today - LaTeX Tutorial
- Syllabus update - Assignment Solutions

Cases If P then Q

If P then P_1 or P_2

If P_1 then Q

If P_2 then Q

Theorem. If $a+b \geq 16$, then either $a \geq 8$ or $b \geq 8$.

$a=17,\ b=1 \quad a=1,\ b=17 \quad a=9,\ b=9$

Proof. Use Cases. Either $a \geq 8$, or $a < 8$.

Case 1: $a \geq 8$

Case 2: $a<8$, $a+b \geq 16$, Therefore $b \geq 8$

$-a > 8 \quad -a \geq 8$

$\square$

Case 1. x has 1 friend 4 strangers

Case 2. x has 2 friends 3 strangers

⋮ could also work

Theorem. In every group of 6 people, there are either 3 mutual friends, or 3 mutual strangers.

Proof. Use cases. Pick a person x. Either they have atleast 3 friends, or 3 strangers.
(1) (2)

Case 1. All of x's friends are mutual strangers, or not.
(1a) (1b)

Case 1a 3 mutual strangers

Case 1b y+z are friends, y+z are friends with x. → 3 mutual friends.

Case 2. All x's strangers are mutual friends or not. → 2b
(2a)

Case 2a. 3 mutual friends

Case 2b. y+z are strangers, y+z strangers with x, → 3 mutual strangers

Contrapositive If P then Q → If not Q then not P.

Theorem. If $a+b \geq 16$, then either $a \geq 8$ or $b \geq 8$.

Proof. Use Contrapositive. If $a<8$ and $b<8$, then $a+b<16$.

$\square$

Theorem. If r is irrational, so is $\sqrt{r}$.

↳ $r \neq \frac{m}{n}$ for every m, n integers

Proof. Use contrapositive. If $\sqrt{r}$ is rational, then r is rational.

If $\sqrt{r}$ is rational, $\sqrt{r} = \frac{m}{n}$, m, n are integers. Therefore, $r = \frac{m^2}{n^2}$, m^2, n^2 are integers, r is rational.

$\square$

Converse - If Q then P not the same as If P then Q

Iff - If and only if, If P then Q, and if Q then P

Theorem. For $a, b \geq 0$, $\frac{a+b}{2} = \sqrt{ab}$ iff $a=b$.

Proof. If $a=b$, then $\frac{a+b}{2} = \sqrt{ab} = a$

If $\frac{a+b}{2} = \sqrt{ab} \xrightarrow{\text{(implies)}} \frac{(a+b)^2}{4} = ab$

$\rightarrow a^2+2ab+b^2 = 4ab \rightarrow a^2-2ab+b^2=0$

$\rightarrow (a-b)^2 = 0 \rightarrow a-b=0 \rightarrow a=b$.

$\square$

Can also do a chain of iffs. (replace → w/ ↔)

Def. x is even if $\frac{x}{2}$ is an integer.

Def x is odd if $\frac{x-1}{2}$ is an integer.

Thm. For integers x, x is even iff x^2 is even

Proof. We'll show if x is even, then x^2 is even.

If x is even, then $\frac{x}{2} = k$, k is an integer. Therefore $\frac{x^2}{4} = k^2$, therefore $\frac{x^2}{2} = 2k^2$

$2k^2$ is an integer, therefore x^2 is even.

If x^2 is even, then x is even.

Use contrapositive → If x is odd, then x^2 is odd.

If x is odd, then $\frac{x-1}{2} = k$, where k is an integer, Therefore $\frac{(x-1)^2}{4} = k^2$,

$\frac{x^2+1-2x}{2} = 2k^2$, therefore $\frac{x^2-1}{2} = 2k^2+x-1$

$2k^2+x-1$ is an integer, therefore x^2 is odd.

$\square$