January 6

Monday, January 6, 2020 1:28 PM

Comp Sci 212- Mathematics for Computer Science

1. Administrivia
2. Propositions
3. Direct Proofs

<u>This Class</u>	<u>Future</u>
Induction	Recursion
Combinatorics	Lower Bounds
Probability	Randomized Algorithms
Graph Theory	Graph Algorithms
Number Theory	Cryptography

Def. <u>Proposition</u> a statement that is true or false

Ex. $2+2=4$ (True)

Ex $2+2=5$ (False)

Ex. For all functions $f(x), g(x)$ (Product Rule)

$$\frac{d}{dx}f(x)g(x) = \left(\frac{d}{dx}f(x)\right)g(x) + \left(\frac{d}{dx}g(x)\right)f(x)$$

$$= \left(\frac{d}{dx}f(x)\right)\left(\frac{d}{dx}g(x)\right) \text{ (False)}$$

Ex. For every non-negative integer n, $(n \in \mathbb{N})$ (False)
n^2+n+41 is prime
↳ only divisible by 1 & itself.

n	n^2+n+41
1	43
2	47
3	53
⋮	⋮
20	461
⋮	⋮
39	1601
40	$1681 = 41^2$

Ex. For all non-negative integers a, b, c, d
$a, b, c, d \in \mathbb{N}$

$a^4+b^4+c^4 \neq d^4$ [Euler's conjecture]

$a = 95,800$ (Noam Elkies)
$b = 217,519$
$c = 414,560$
$d = 422,481$

Def. Predicate a proposition that depends on a variable

Ex. n is a perfect square

$P(n) =$ "n is a perfect square"

$P(4), P(9) =$ True $P(5), P(7) =$ False

<u>Axiom</u> a proposition that is accepted as being true (does not need to be proved).

This Class- use your judgement

Theorem, Lemma, Claim

<u>Direct proofs</u> Def. <u>Implication</u> a statement of the form If P then Q

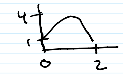
Def. Direct proof a sequence of implications that each do not have to be proved, combine to form its own implication.

"If P_1, then P_2. Therefore P_3. Therefore P_4"

If P_1, then P_4

Thm. If $0 \le x \le 2$ then $f(x) = -x^3+4x+1 \ge 0$.

x	$f(x)$
0	1
0.5	2.875
1	4
1.5	3.625
2	1



$f(x) \ge 1$

$-x^3+4x \ge 0$

$-x(x+2)(x-2)$

neg pos neg

Proof If $0 \le x \le 2$, then $-x \le 0$, $x+2 \ge 0$, $x-2 \le 0$.
Therefore, $-x(x-2)(x+2) \ge 0$.
By adding 1 to both sides, $-x^3+4x+1 \ge 1$.
Therefore, $-x^3+4x+1 \ge 0$ □

Thm. (Arithmetic mean- Geometric mean Inequality). If $a, b \ge 0$, then

$$\frac{a+b}{2} \ge \sqrt{ab}.$$

Ex. $a=2, b=2$ $\frac{a+b}{2} = 2$, $\sqrt{ab} = 2$

$a=1, b=9$ $\frac{a+b}{2} = 5$ $\sqrt{ab} = 3$

Proof. If $a, b \ge 0$, $(a-b)^2 \ge 0$, or
$a^2-2ab+b^2 \ge 0$.
By adding $4ab$ to both sides,
$a^2+2ab+b^2 \ge 4ab$.
Therefore, $(a+b)^2 \ge 4ab$.
Therefore by taking the square root of both sides, $a+b \ge 2\sqrt{ab}$, or $\frac{a+b}{2} \ge \sqrt{ab}$.

Examples are not proofs.
(Counterexamples are)